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Modelling Turbulence in the Lower Atmosphere Using Richardson's Criterion

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It is assumed that the presence of turbulence in the atmosphere is strongly correlated with low values of the gradient Richardson number ($Ri \leq 1.0$). The critical Richardson number $Ri_c = 1.0$, is an indicator of the onset of turbulence. Values of the gradient Richardson number are calculated using rawinsonde measurements of wind and temperature obtained from 144 rawinsonde stations over a period from 1971-1975. Computer algorithms are developed for a statistical analysis of the critical Richardson number based on years, latitude, longitude, altitude, and season. Twice daily sets of

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rawinsonde height measurements of wind and temperature are fit using a Hermite interpolation algorithm. Richardson numbers (Ri) calculated from these profiles are analyzed to determine the number and percent of occurrences of $Ri \leq Ri_c$ on a seasonal basis. Summary (seasonal) statistics on this critical Richardson number, or turbulence indicator, as well as all variables used, are computed for 1 km altitude bins from 2 to 25 km, a range determined to provide good statistical accuracy and precision. Particular features in the height profiles of percent occurrences of $Ri \leq Ri_c$ appear with remarkable consistency and are dependent on season and latitude. A characteristic peak layer in the percent occurrence of $Ri \leq 1.0$ near 10 km is found in data obtained from sites located from low to mid-latitudes. The range in occurrence for this layer is from 40% in winter to 5% in summer. Multiple regression analyses are applied using data from 21 of the stations covering a region from latitude 25°N to 48°N and longitude 69°W to 123°W. The data are sectioned for separate analysis into four altitude regions 2 to 7 km, 8 to 13 km, 14 to 19 km, and 20 to 25 km. The analyses demonstrate that the patterns of turbulence based on a critical Richardson number of 1 have a substantial component which is stable and reproducible from year to year. The regressions relate percent occurrences of $Ri \leq Ri_c$ to location (latitude, longitude), altitude, and season; the coefficients of multiple correlation for altitudes below 20 km range from 0.62 to 0.78 (that is, up to 60% variation in percent occurrences explained). Yearly variations increases the square of the multiple correlation coefficients only by a maximum of 0.003 (that is, at most 3% variation explained).

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1. INTRODUCTION

Elaborate and expensive experimental techniques are frequently used to obtain measurements for atmospheric turbulence studies. A common approach to turbulence investigation is to obtain accurate measurements of wind velocity fluctuations for spectrum analysis. There are, however, many subtle problems involved in the practical analysis of finite lengths of data. In the case of finite lengths of periodic data where the fundamental wavelength is known and digitization is performed properly, there are no problems associated with interpretation of the resultant spectra.[1,2] This is not the case with turbulence; as pointed out by Hinze,[3] in his Introduction; "Turbulence, consists of many superimposed quasi-periodic motions. Though a harmonic analysis of the velocity fluctuations can be carried out, this fact is no proof that, conversely, the turbulent fluctuations are composed of these harmonics."

The approach we have used here consists of a statistical analysis performed on a large data base of twice daily rawinsonde measurements of wind and temperature in the altitude

(Received for publication 14 December 1981)

1. Oppenheim, A.V., and Schafer, R.W. (1975) Digital Signal Processing, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
2. Brigham, E.A. (1974) The Fast Fourier Transform, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
3. Hinze, J.O. (1959) Turbulence, McGraw-Hill, Inc., New York, N.Y.

range 2 to 25 km. The Richardson criterion (Section 2) is applied to daily data and the likelihood or relative frequency of occurrence of turbulence based on a critical Richardson number of one ($Ri_c = 1$) is determined on a seasonal basis. A similar study of Richardson numbers in the mesosphere using rocket grenade data has been reported by Zimmerman and Murphy.[4] These data, however, were only available at six locations and the number of measurements obtained were only suitable, statistically, for determining semiannual variations. Rawinsonde data used in this study have the advantage that they are available on magnetic tape from many stations at low cost (140 locations of which 90% are in the Americas). Also, data collection techniques are standardized and all data are edited at a central location; the National Climatic Center. Seasonal height profiles of the occurrences of turbulence based on a critical Richardson number of 1 have been qualitatively evaluated at many of these stations. These height profiles revealed interesting structure and variability with altitude, latitude, and season. These features will be presented in this report along with a regression analysis performed on data from 21 of the stations located within the continental United States. The objective was to determine the significance of a relationship between the proportion of Richardson numbers less than unity (turbulence measure), and altitude, location, and season.

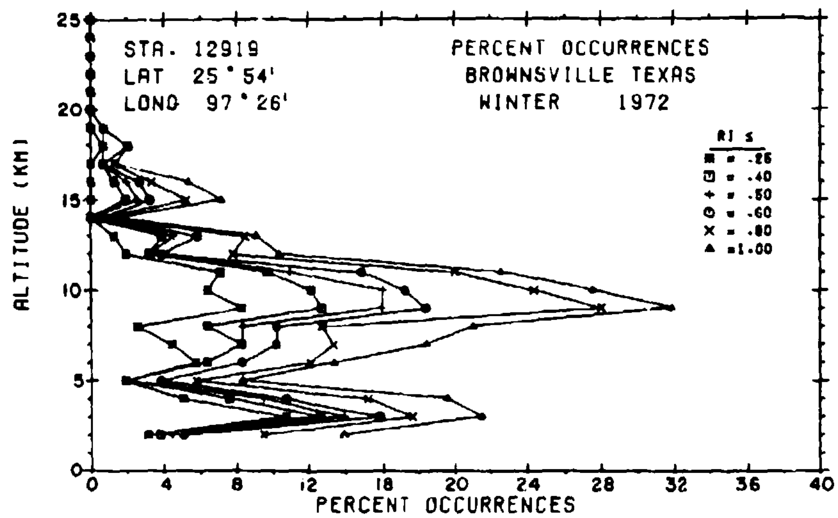

Figure 1. Occurrences of Turbulence. The percent occurrences are obtained by taking the ratio of the total number of occurrences of $Ri \leq Ri_c$ to the total number of measurements at a given altitude. This is repeated for Ri_c ranging in steps from 0.25 to 1.00

4. Zimmerman, S.P., and Murphy, E.A. (1977) Stratospheric and mesospheric turbulence, Dynamical and Chemical Coupling, B. Grandal and J.A. Holtet, Eds., D. Reidel Publishing Co., Dordrecht, Holland, AFGL-TR-78-0020, AD A050128.

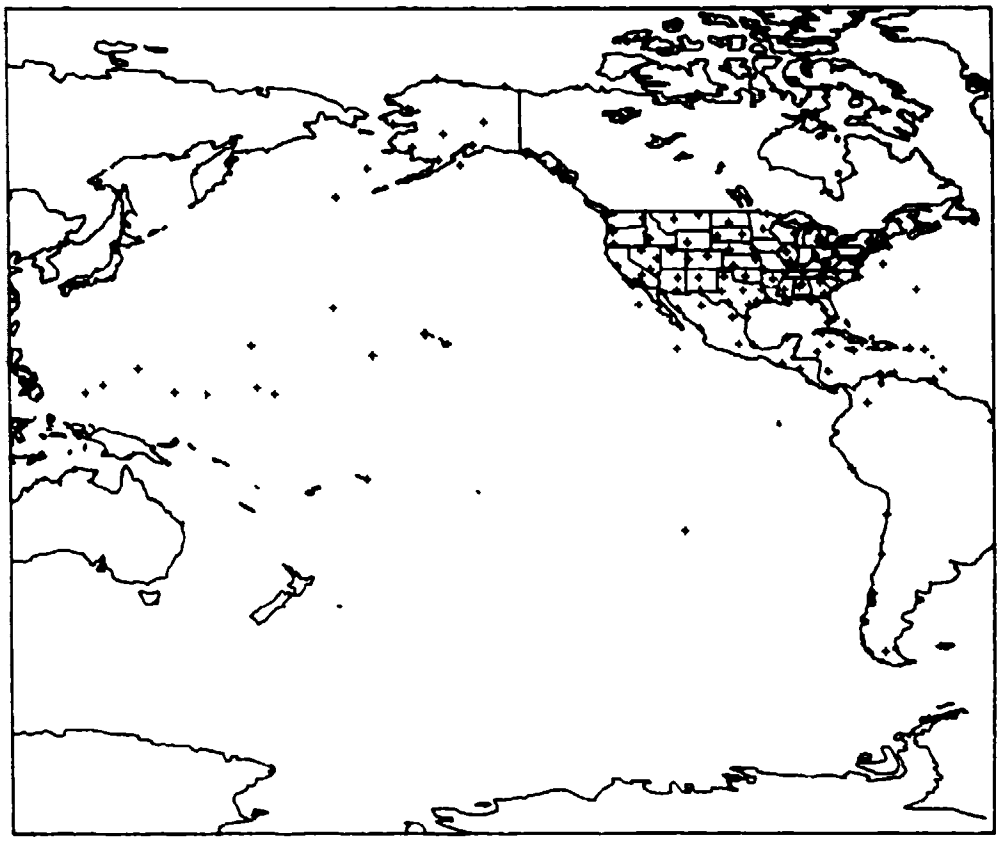

Figure 2. Location of the 140 Available Rawinsonde Reporting Stations

Table 1. Rawinsc ıde Data Collection Stations

Name	Latitude		Longitude	
ATLANTIC STATION H	38	00N	071	00W
SAN DIEGO, CA	32	49N	117	08W
YUCCA FLAT, NV	36	57N	116	03W
SAN NICOLAS, CA	33	16N	119	33W
HUNTINGTON, WV	38	22N	082	33W
SALEM, IL	38	39N	088	58W
CENTREVILLE, AL	32	54N	87	15W
LAKE CHARLES, LA	30	07N	093	13W
JACKSON, MS	32	19N	909	05W
MONETT, MO	36	53N	093	54W
LONGVIEW, TX	32	21N	094	39W
BOGOTA, COLUMBIA SA	04	42N	074	90W
BARBADOS, WI	13	04N	059	30W
SANTO DOMINGO, DOM. REP.	18	28N	069	53W

Table 1. Rawinsonde Data Collection Stations (Cont)

Name	Latitude		Longitude	
TRINIDAD, WI	10	35N	061	21W
SAN JUAN, PR	18	26N	066	00W
WILLEMSTAD, CURACAO N.A.	12	11N	058	58W
ST. MAARTEN, N.A.	18	03N	063	07W
GUANTANAMO, CUBA NAS	19	54N	075	09W
KINGSTON, JAMAICA W I	17	56N	076	47W
SWAN ISLAND	17	24N	083	56W
GRAND CAYMAN, W I	10	18N	081	22W
SAN ANDRES, COLOMBIA	12	35N	081	42W
CHOLUTECA, HONDURAS	13	18N	087	11W
GUATEMALA CITY, GUATEMALA	14	32N	090	34W
MEXICO D F I A	19	26N	099	04W
VERACRUZ, MEXICO	19	10N	096	07W
APALACHICOLA, FL	29	44N	085	02W
MIAMI, FL	25	48N	080	16W
TAMPA BAY, FL	27	42N	082	24W
KEY WEST, FL	24	35N	081	41W
MERIDA YUCATAN, MEXICO	20	57N	089	39W
BOOTHVILLE, LA	29	20N	089	24W
VICTORIA, TX	28	51N	096	55W
BROWNSVILLE, TX	25	54N	097	26W
ST. GEORGE, BERMUDA	32	22N	064	41W
GREENSBORO, NC	36	05N	079	57W
DAYTON, OH	39	52N	084	07W
WAYCROSS, GA	31	15W	082	24W
ATHENS, GA	33	57N	083	19W
CHARLESTON, SC	32	54N	080	02W
NASHVILLE, TN	36	15N	036	34W
STEPHENVILLE, TX	32	13N	098	11W
LITTLE ROCK, AR	34	50N	092	16W
OKLAHOMA CITY, OK	35	24N	097	36W
DODGE CITY, KS	37	46N	099	58W
TOPEKA, KS	39	04N	095	37W
CARIBOU, ME	46	52N	068	01W
CHATHAM, MA	41	40N	069	58W
BUFFALO, NY	42	56N	078	44W
ALBANY, NY	42	45N	073	48W
PORTLAND, ME	43	39N	070	19W
FLINT, MI	42	58N	083	44W
PEORIA, IL	40	40N	089	41W
SAULT STE MARIE, MI	46	28N	084	22W
GREEN BAY, WI	44	29N	088	08W
INTERNATIONAL FALLS, MN	48	34N	093	23W
ST. CLOUD, MN	45	33N	094	05W
HURON, SD	33	12N	098	13W
KEFLAVIK, ICELAND FWF	63	58N	022	36W
COCORRO IS., MEXICO	18	14N	111	03W
HILO, HI	19	43N	155	04W
JOHNSTON ISLAND	16	44N	169	31W
CHIHUAHUA, MEXICO	28	42N	106	04W
MAZATLAN, MEXICO	23	11N	106	25W
DEL RIO, TX	29	22N	100	55W
MONTERREY, MEXICO	25	52N	100	12W
EMPALME, MEXICO	27	57N	110	48W

Table 1. Rawinsonde Data Collection Stations (Cont)

Name	Latitude		Longitude	
GUADALUPE IS., MEXICO	28	53N	118	18W
LIHUE KAUAI, HI	21	59N	159	21W
BARKING SANDS, HI	22	02N	159	47W
MIDWAY ISLAND, NS	28	13N	177	21W
MIDLAND, TX	31	56N	102	12W
EL PASO, TX	31	49N	106	24W
AMARILLO, TX	35	14N	101	42W
ALBUQUERQUE, NM	35	03N	106	37W
DENVER, CO	39	46N	104	53W
GRAND JUNCTION, CO	39	07N	108	32W
ELY, NV	39	17N	114	51W
TUCSON, AZ	32	07N	110	56W
WINSLOW, AZ	35	01N	110	44W
OAKLAND, CA	37	45N	122	13W
BISMARCK, ND	46	46N	100	45W
LANDER, WY	42	49N	108	44W
NORTHPLATTE, NE	41	03N	100	41W
RAPID CITY, SD	44	03N	103	04W
SALT LAKE CITY, UT	40	46N	111	58W
WINNEMUCCA, NV	40	54N	117	48W
BOISE, ID	43	34N	116	13W
GREAT FALLS, MT	47	29N	111	21W
SPOKANE, WA	47	38N	117	32W
MEDFORD, OR	42	22N	122	52W
SALEM, OR	44	55N	123	01W
ANNETTE, AK	55	02N	131	34W
YAKUTAT, AK	59	31N	139	40W
KODIAK, AK	57	45N	152	29W
KING SALMON, AK	58	41N	156	39W
COLD BAY, AK	55	12N	162	43W
ADAK, AK	51	53N	176	39W
ST. PAUL IS., AK	57	09N	170	13W
ANCHORAGE, AK	61	10N	150	01W
FAIRBANKS, AK	64	49N	147	52W
MCGRATH, AK	62	58N	155	37W
BETHEL, AK	50	47N	161	48W
KOTZEBUE, AK	66	52N	162	38W
NOME, AK	64	30N	165	26W
BARTER ISLAND, AK	70	08N	143	38W
BARROW, AK	71	18N	156	47W
YAP CAROLINE IS.	09	29N	138	05E
KOROR CAROLINE IS.	07	20N	134	29E
PONAPE CAROLINE IS.	06	58N	158	13E
TRUK, CAROLINE IS.	07	28N	151	51E
KWAJALEIN ATOLL MI PMP WS	08	44N	167	44E
MAJURO, MARSHALL IS.	07	05N	171	23E
GUAM TAGUAC MI	13	33N	144	50E
WAKE ISLAND	19	17N	166	39E
LIMA, PERU	12	01S	077	08W
ANTOFAGASTA, CHILE	23	25S	070	28W
QUINTERO, CHILE	32	47S	071	32W
PUERTO MONIT, CHILE EL TEMPUAL	41	26S	073	07W
PUNTA ARENAS, CHILE	53	02S	070	51W
PAGO PAGO, AMERICAN SAMOA	14	20S	170	43W

Table 1. Rawinsonde Data Collection Stations (Cont)

Name	Latitude		Longitude	
EASTER ISLAND, CHILE	27	10S	109	26W
BYRD STA, ANTARCTIC	80	01S	119	31W
DIEGO GARCIA	07	18S	072	24E
MCMURDO SOUND ANTARCTIC	77	53S	166	44E
HALLETT ANTARCTIC	72	19S	170	13E
AMUNDSEN SCOTT ANTARCTIC				
CHINA LAKE, CA	35	41N	117	41W
PT MUGU, CA	34	06N	119	07W
SAN NICOLAS, CA PMR WS SITE 1	33	14N	119	27W
VANDENBERG AFB, CA	34	45N	120	34W
CAPE HATTERAS, NC	35	16N	075	33W
STERLING, VA	38	59N	077	28W
WALLOPS IS., VA	37	51N	075	29W
GLASGOW, MT	48	13N	106	37W
QUILLAYUTE, WA	47	57N	124	33W
NEW YORK, NY	40	47N	073	46W
PITTSBURGH, PA	40	32N	080	14W
OMAHA, NE	41	22N	096	01W

2. THE RICHARDSON CRITERION

Here we will make use of the criteria for stability developed by Richardson.[5] Consider his energy budget equation for a large volume of the atmosphere (where he has assumed that the eddy viscosity μ = eddy conductivity c):

$$\frac{\partial E'}{\partial t} - \frac{2\mu\pi}{\rho} E' = \int\int c \left[\left(\frac{\partial \bar{v}_x}{\partial Z}\right)^2 + \left(\frac{\partial \bar{v}_y}{\partial Z}\right)^2 - \frac{g}{C_p}\frac{\partial \sigma}{\partial Z} \right] dZ\, dA \,. \tag{1}$$

The second term on the left side is the dissipation due to viscosity and the terms on the right, the supply from the mean wind and the loss from convective activity. With the assumption that the eddy viscosity μ equals the eddy conductivity c, Richardson considered a special application of Eq. (1) where the volume of atmosphere is initially at rest then set into motion by an approaching depression so that c and E' are just somewhat greater than zero. Under these conditions with the dissipation term negligibly small, then E' will increase if

$$\left(\frac{\partial \bar{v}_x}{\partial Z}\right)^2 + \left(\frac{\partial \bar{v}_y}{\partial Z}\right)^2 > \frac{g}{C_p}\frac{\partial \sigma}{\partial Z} \,. \tag{2}$$

5. Richardson, L. B. (1920) The supply of energy from and to atmospheric eddies, Proc. Roy. Soc. A. 97:354-373.

where $\frac{\partial \sigma}{\partial Z}$ is the entropy gradient.

From the first law of thermodynamics

$$dq = dE + PdV \ . \tag{3}$$

Since

$$dE = \left(\frac{\partial E}{\partial V}\right)_T dV + \left(\frac{\partial E}{\partial T}\right)_V dT \tag{4}$$

and the internal energy for an ideal gas is zero, then

$$dE = \left(\frac{\partial E}{\partial T}\right)_V dT = C_v \, dT \tag{5}$$

and

$$dq = C_v \, dT + PdV \ . \tag{6}$$

Differentiating the gas equation

$$PdV + VdP = RdT \tag{7}$$

and substituting into Eq. (6) for PdV, results in the following form of the first law:

$$dq = (C_v + R) \, dT - VdP \ . \tag{8}$$

Since

$$C_p - C_v = R \tag{9}$$

then

$$dq = C_p \, dT - VdP \ . \tag{10}$$

Substituting

$$V = RT/P \tag{11}$$

and dividing by the integrating factor T to obtain an exact differential, the change in entropy is defined as

$$d\sigma = \frac{dq}{T} = C_p \frac{dT}{T} - R\frac{dP}{P} . \qquad (12)$$

Substituting from the hydrostatic and gas equations, the entropy gradient can be expressed in the form:

$$\frac{d\sigma}{dZ} = \frac{C_p \, dT}{T \, dZ} + \frac{g}{T} . \qquad (13)$$

The right side of Eq. (2) can then be expressed as a function of T and Z and

$$\frac{g}{C_p}\frac{d\sigma}{dZ} = \frac{g}{T}\left[\frac{dT}{dZ} + \frac{g}{C_p}\right] . \qquad (14)$$

The constant $\frac{g}{C_p} = \left(-\frac{dT}{dZ}\right)_{\text{adiabatic}}$ can be obtained from a differentiated form of Poisson's equation with the potential temperature, θ, independent of altitude.

With the definition $\Gamma_d = \frac{g}{C_p} = \left(\frac{-dT}{dZ}\right)_{\text{adiabatic}}$, the dry adiabatic lapse rate, then the original criteria, Eq. (2), can be written as a ratio

$$\frac{\frac{g}{T}\left[\frac{dT}{dZ} + \Gamma_d\right]}{\left(\frac{\partial \bar{v}_x}{\partial Z}\right)^2 + \left(\frac{\partial \bar{v}_y}{\partial Z}\right)^2} < 1 . \qquad (15)$$

Also, logarithmic differentiation of Poisson's equation with the potential temperature varying with height yields the relationship:

$$\frac{1}{\theta}\frac{\partial \theta}{\partial Z} = \frac{1}{T}\left[\frac{dT}{dZ} + \frac{g}{C_p}\right] . \qquad (16)$$

Except for a factor g, this is equivalent to the numerator in Eq. (15) and yields the other familiar form of the criteria:

$$\frac{\frac{g}{\theta}\frac{\partial \theta}{\partial Z}}{\left(\frac{\partial \bar{v}_x}{\partial Z}\right)^2 + \left(\frac{\partial \bar{v}_y}{\partial Z}\right)^2} < 1 . \qquad (17$$

A form of the stability criterion was first used and named after Richardson by Paeschke.[6] Swinbank[7] was somewhat critical of Richardson's earlier work on the conditions for the onset of turbulence. In a heated reply by Richardson to Swinbank which follows Swinbank's paper, Richardson mentions his embarrassment at having to note Paeschke's failure to take into account the veer of the wind with height, especially since Paeschke had named the dimensionless quantity after him. The reply by Richardson contains a number of interesting comments on his 1920 paper and also mention of his further explication of that work in Richardson.[8]

The Richardson number can be thought of as the ratio of the destruction of turbulence by stable temperature gradients to the production of turbulence by wind shear. Thus turbulence can occur only for $Ri < 1.0$. This criterion is now widely used, although laboratory tank experiments by Thorpe[9] have indicated that the onset of instability does not occur above a value of 0.25. Also, theoretical studies by Miles and Howard[10] suggest that $Ri_c \leq 0.25$ is a necessary but not sufficient condition for the onset of turbulence.

Considering the flux Richardson number

$$Ri_f = \frac{K_h}{K_m} Ri ,$$

where K_h and K_m are the turbulent coefficients for heat and momentum, there may be some critical value of $Ri_{fc} < 1$ where instability will occur. Perhaps, as pointed out by Vinnichenko et al,[11] under certain conditions in the free atmosphere, either K_h or K_m could be the larger. It is then possible that instability occurs when $Ri > 1$. In this report it is assumed that $Ri_c \leq 1$ indicates the presence of turbulence. In Figure 1, the frequency of occurrence of Ri less than or equal to 0.25, 0.40, 0.50, 0.60, 0.80, and 1.0 is plotted as a function of height for Brownsville, Texas.

6. Paeschke, W. (1938) Experimentelle Untersuchongen zum Rauhigkeits und Stabilitats problem in der bondennanen Luftschicht, Beitrange zur Physik der Atmosphare (contributions to Atmospheric Physics), 24:163-189.

7. Swinbank, W.C. (1952) The criterion of atmospheric turbulence, Quart. J. Roy. Meteor. Soc. 78:420-425.

8. Richardson, L.F. (1925) Turbulence and vertical temperature difference near trees, Phil. Mag. S. 6:49, 81-89.

9. Thorpe, S.A. (1968) A method of producing a shear flow in a stratified fluid, J. Fluid. 32:693-704.

10. Miles, J.W., and Howard, L.N. (1964) Note on a heterogeneous flow, J. Fluid Mech. 20:331-336.

11. Vinnichenko, N.K., Pinos, N.Z., Schimeter, S.M., and Shur, G.N. (1973) Turbulence in the Free Atmosphere, Central Aerological Observatory, Observatory, Dolgoprudny, USSR. (Translated from Russian by John A. Dutton, Pennsylvania State University for the Consultants Bureau, New York.)

These height profiles have similar structures; however, there are roughly three times more occurrences of Ri $\leq$ 1.0 than Ri $\leq$ 0.25. Data from several stations were examined in this manner and yield similar results. The use of $Ri_c \leq 1.0$ provides a larger data base for statistical analyses.

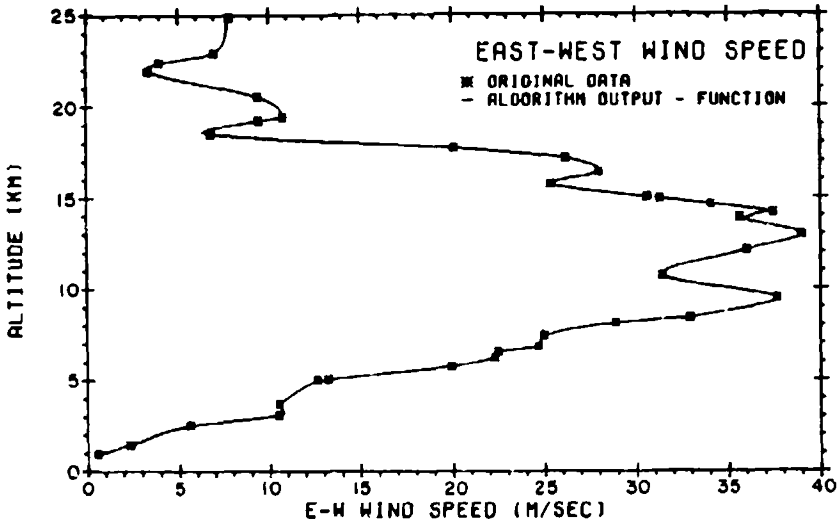

Figure 3. Frequency of Occurrences of Ri. The fitting algorithm provides a smooth fit (solid line) through the original east-west component data obtained on 16 February 1975 at Brownsville, Texas

3. DATA BASE

The rawinsonde data used in this study have been provided on magnetic tape by the National Climatic Center. These consist of twice daily "upper air observations" of winds and temperatures taken at 0000 and 1200 hours GMT. Data from 140 stations (mainly from the United States and South America) for the years 1971-1975 (Figure 2, Table 1) have been reformatted for computer processing to provide sample statistics for the Richardson number study.

The rawinsonde system was implemented to provide data on a global basis at standard millibar levels (mandatory levels) to obtain charts and maps for weather analysis. A calibrated baroswitch is used to measure pressure levels and a hygristor and thermistor are used for the measurement of relative humidity and temperature. A transmitted signal from the balloon package is tracked automatically to determine the azimuth and elevation. Altitude and winds are obtained by

using this information along with the geopotential height. Hess[12] has shown that the pressure at a geopotential height can be determined to within 1 mb by using a simple barometric element of the type used in the rawinsonde system.

During rawinsonde measurements a person in the data recording loop records significant temperature levels between mandatory levels if the slope of the traces on the recorder record changes by a small but detectable amount. Significant temperature levels are recorded if a departure from the trend between successive levels represents a change greater than ± 1°C from the surface up to 100 mb (approximately 16 km) and greater than ± 2°C up to the level of termination. The balloon rise rate is about 300 m per min and position data is obtained at 1-min intervals. The average component wind speeds are obtained from the position data by using a 2-min overlapping interval below 14 km and a 4-min overlapping interval at higher altitudes. This approach results in wind averaging layers of approximately 0.6 km below 14 km and 1.2 km above 14 km. A compilation of standard error estimates for meteorological data and a list of references dealing with the subject of errors in meteorological data has been published by the Meteorological Group Range Commander's Council.[13] In the present report it is assumed that there are no bias errors in the data. Also random errors are minimized by grouping the data into three-month seasons to provide the largest number of sets of measurements. This number ranges, on the average, from 180 at the 2 km level to 100 at the 25 km level. The number of measurements decreases as altitude increases because of balloon bursts.

The data for each launch were read and verified for completeness (Conversion to standard units was done on pressure, altitude, and temperature). To obtain temperatures and gradients of temperatures and of wind components required to determine the Richardson number at the 1-km levels used in this study, the vertical profiles of wind and temperature were fit using a standard cubic Hermite interpolation algorithm,[14] programmed by Tsipouras and Cormier.[15] This algorithm provides a smooth fit passing through all the data points and allows the computerized determination of the fitting function and its derivative at any point while avoiding problems associated with discontinuities encountered in linear fits (that is, a first

12. Hess, S.L. (1959) Introduction to Theoretical Meteorology, Henry Holt and Company, New York, N.Y.

13. Meteorological Group Range Commander's Council, 1977: Meteorological Data Error Estimates (Document 110-77, Secretarial Range Commander's Council, White Sands Missile Range, New Mexico 88002.

14. Forsythe, G., Malcolm, M. (1977) Computer Methods for Mathematical Computations, Prentice Hall, Inc., Englewood Cliffs, New Jersey.

15. Tsipouras, P., and Cormier, R.F. (1973) Hermite Interpolation Algorithm for Constructing Reasonable Analytic Curves Through Discrete Data Points, AFCRL-TR-73-0400, AD 768869.

order hold). In Figure 3, the original east-west wind speed component data from a typical set of measurements are overlaid (continuous curve) with the output of the fitting algorithm. It should be noted that there is substantial resolution in the original data, usually over one data point per km, implying that the fits should be accurate. Also, the 1-km intervals chosen are consistent with the original data rate.

In Figure 4, the straight lines connect at points which represent the first differences in the original wind component data of Figure 3, at altitudes taken at the midpoints between successive data points. The dots in Figure 4 are the output of the wind shear algorithm of the fitting function. This indicates that the derivative of the fitting function is a good approximation to the average gradient as obtained from the first differences of the wind component data. The vertical marks at the bottom of Figure 4 represent altitude levels at which the original data have been obtained.

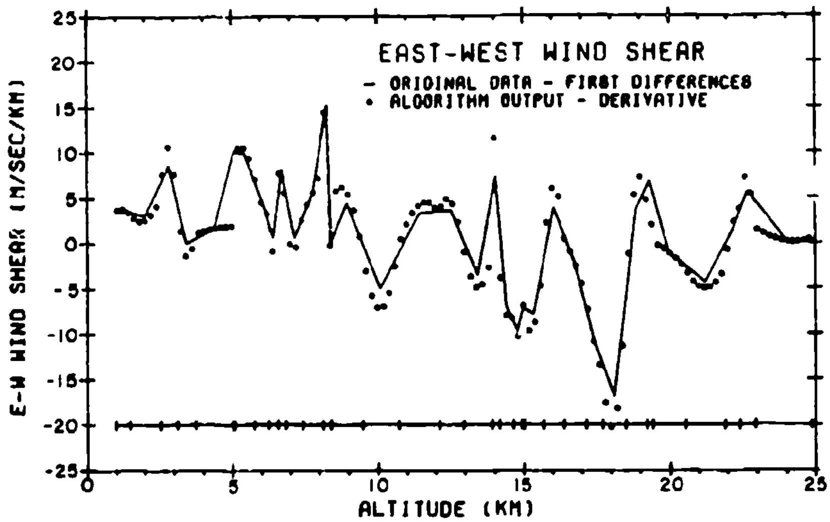

Figure 4. Differences in Wind Component Data. The solid line follows the average gradient obtained from the first differences of the data in Figure 3 as a function of the average altitude between data points. The dots represent the output of the derivative of the fitting function of the E-W wind shear

Initially, valid points from each launch were put into altitude bins defined as the following: 2-7.5 km, 7.5-13.5 km, 13.5-20.5 km, and 20.5-25 km. (The reason for this bin structure will be apparent later.)

To determine the average resolution of the data, the average distance between each point was calculated and compiled over a 5-yr seasonal average. Results show:

Altitude Bin	Average Distance Between Readings
2-7.5 km	0.5 km
7.5-13.5 km	0.7 km
13.5-20.5 km	0.85 km
20.5-25 km	1.2 km

This demonstrates that data collection was consistent (average distance between readings falls within one standard deviation of their mean) and interpolation to whole kilometers was appropriate. Variables computed for 1-km intervals from 2 to 25 km include wind shear, density, temperature gradient, Brunt-Väisälä frequency, turbulent intensity, rate of dissipation of kinetic energy, turbulent diffusivity, and the Richardson number.

Data were grouped seasonally to assure statistical stability. Seasons were separated using the solstice and equinox dates. There were from 100 to 1[illegible]0 measurements at any given altitude between 2 km and 25 km.

The Richardson number is computed using the temperatures and wind component values along with their derivatives. Percent occurrences of Ri's less than or equal to unity were computed for each season and year at each altitude. Values less than or equal to 1 are used as an indicator of the onset of turbulence. For each station there are 980 data readings (7 years × 4 seasons per year × 35 altitude bins).

A representative set of 21 stations (Table 2) were chosen from the [illegible] stations in the continental United States (Figure 5). They were chosen for a wide range of latitudes and longitudes spanning the country. Also, sites close to each other were included to measure correlation between sites. Topography was another consideration. It was desirable to include stations with varying physical configurations to get a general representation of the country. Years 1971-1975 were chosen since the data were the most recent at the time and most of the sites were reporting on a regular basis through this period. Also stations were chosen on the basis that they were using similar and up-to-date equipment.

The number of samples at each kilometer bin was recorded for the 21 stations. A minimum of 100 samples was required for statistical accuracy. Table 3 lists the highest altitude bin that attains a minimum of 100 samples. Over 85% of the groups have 100 samples at 25 km. Due to the geographic formations surrounding some stations, balloon readings began at 2 km. The altitude range 2 to 25 km was used for analysis.

Table 2. Stations Used in Multiple Regression Analysis

Station Location	Latitude	Longitude
Brownsville, TX	25° 54'	97° 26'
Chatham, MA	41° 40'	69° 58'
Dayton, OH	39° 52'	84° 07'
Denver, CO	39° 46'	104° 53'
Flint, MI	42° 58'	83° 44'
Glasgow, MT	48° 13'	106° 37'
Green Bay, WI	44° 29'	88° 08'
Greenboro, NC	36° 05'	79° 57'
International Falls, MN	48° 34'	93° 23'
Medford, OR	42° 22'	122° 52'
Miami, FL	25° 48'	80° 16'
North Platte, NE	41° 08'	100° 41'
Portland, ME	43° 39'	70° 01'
Salem, OR	44° 55'	123° 01'
Saulte Ste. Marie, MI	46° 28'	84° 22'
Spokane, WA	47° 28'	117° 32'
Topeka, WA	39° 04'	95° 37'
Washington, DC	38° 59'	77° 28'
Waycross, GA	31° 15'	82° 24'
Winslow, AZ	35° 01'	110° 44'
Great Falls, MT	47° 29'	111° 21'

Between seasons, there were more fluctuations than within seasons. For example, height profiles of turbulence (Figures 7 through 27) show how turbulence differs seasonally. Winter consistently has the highest Richardson number range (that is, the highest turbulence range) and the greatest fluctuations. Conversely, summer has the lowest Richardson number range (that is, lowest turbulence range).

Thus, the data available has been reduced from 980 data points to a more consistent 480 points, (24 altitude bins × 4 seasons × 5 years).

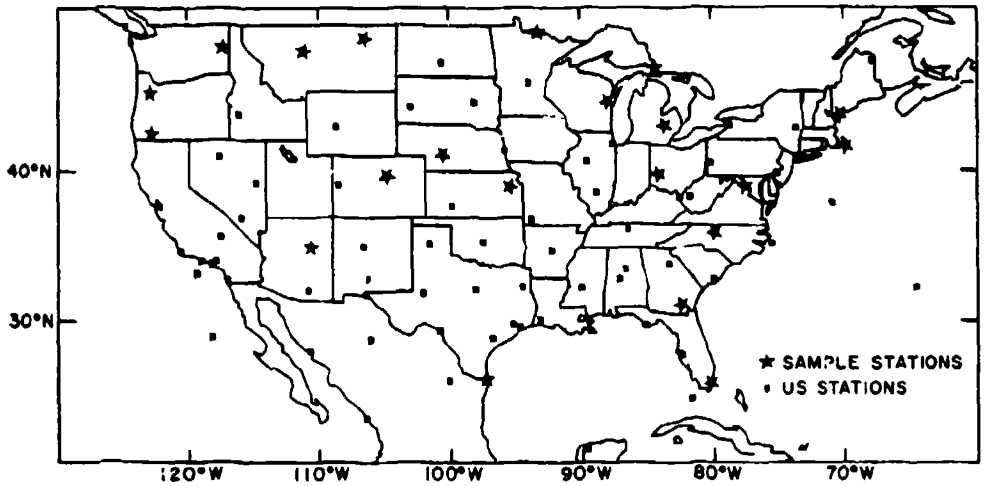

Figure 5. Location of the 81 Available Rawinsonde Stations in the Continental United States. The star marks those stations used in the regression analysis

4. REGRESSION ANALYSIS

Multiple regression techniques were used to investigate the patterns of turbulence over the continental United States.[16] Using this technique, a quantifiable relationship was established between the location variables (latitude and longitude), altitude, seasonal and yearly variation, and the dependent variable percent Ri less than or equal to one. By way of the regression coefficients and t statistics (ratio of regression coefficients to their standard errors), we can quantify the relationship of turbulence to our explanatory variable set.

The patterns in the percent occurrences of $Ri \leq 1.0$ as a turbulence indicator show three peaks or maximums; one below 7 km, one between 8 to 13 km, and one in the 14 to 19 km range. This is apparent from the plot of Brownsville, Texas (Figure 6). In order to account for these peaks in a regression analysis without having to use higher order polynomials in altitude, it was necessary to partition the data set into four altitude bins and analyze each of the regions separately: 2-7 km, 8-13 km, 14-19 km, and 20-25 km.

16. Draper, N., and Smith, H. (1966) Applied Regression Analysis, John Wiley and Sons, Inc., New York, N.Y.

Thus, multiple regression was the analysis used to quantify turbulence for the 21 stations selected in the continental United States. The altitude region 2-25 km is the region of interest; data collection is consistent over this range. The region is divided into four groups: 2-7 km, 8-13 km, 14-19 km, and 20-25 km to capture peaks appearing in each bin.

Table 3. Highest Altitude With Greater Than 100 Samples

	W	SP	SU	F	5-year Average
Brownsville	25	28	27	27	26.75 km
Chatham	23	26	26	25	25
Dayton	24	28	28	26	26.5
Denver	26	28	28	26	27
Flint	23	27	25	26	25.25
Glasgow	28	28	26	24	26.5
Great Falls	24	28	28	24	26
Green Bay	24	21	28	26	24.75
Greensboro	18	26	28	26	24.5
International Falls	26	28	28	26	27
Medford	26	28	29	27	27.5
Miami	27	28	26	26	26.75
North Platte	24	28	28	26	26.5
Portland	18	27	28	26	24.75
Salem	26	28	28	26	27
Sault Ste. Marie	26	28	28	28	27.5
Spokane	26	28	28	26	27
Topeka	24	28	27	24	25.75
Washington, D.C.	26	28	28	27	27.25
Waycross	26	28	28	28	27.5
Winslow	25	26	26	26	25.75
Average	24.5	27.2	27.4	26.0	26.3

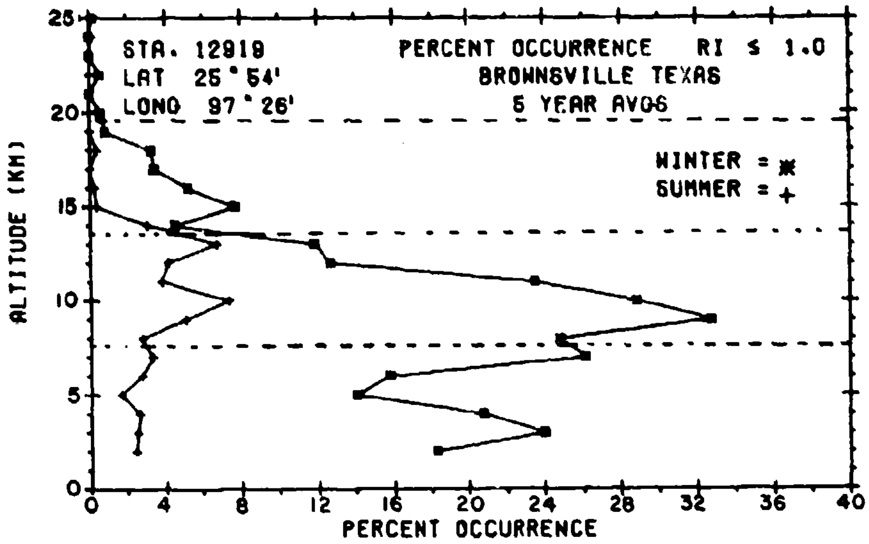

Figure 6. The Troposphere and Stratosphere is Divided Into Four Regions for Separate Regression Analysis. The Brownsville, Texas occurrences have peaks which are typical of the peaks in these regions generally found in these data

In symbols a multiple regression model is written as follows:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots \beta_k X_k + \epsilon$$

where

Y = dependent variable (that is, % Ri $\leq$ 1),

X_i = ith independent variable (four seasons: Winter, Summer, Spring, Fall; $(\text{altitude})^i$, $i = 1, 2, 3$: $(\text{latitude})^j$ $(\text{longitude})^j$ $j = 1, 2$; 5-yearly indicators for 1971, 1972, 1973, 1974, 1975),

β_i = ith regression coefficient.

ϵ = random error term.

Yearly variation was quantified by use of five indicators (zero-one) variables (1971, 1972, 1973, 1974 and 1975). For example:

$$1971 = \begin{cases} 1 \text{ if reading taken in } 1971 \\ 0 \text{ otherwise} \end{cases}$$

The season indicators (winter, spring, summer, and fall) are defined in a similar fashion.

$$\text{Winter} = \begin{cases} 1 \text{ if reading taken in winter} \\ 0 \text{ otherwise} \end{cases}$$

For this study, a stepwise regression analysis is performed. Any variable is a candidate for entry into the model. At the first step, the variable most highly correlated with the dependent variable is entered into the model. Next, the partial F statistics associated with each of the remaining variables are computed. These are based on the regression equation containing that variable and the variable initially selected. The variable with the highest partial F statistic is next entered. This process continues until none of the remaining variables have a significant partial F (that is, at the 0.05 level of significance). A variable that has entered may be removed later if it becomes nonsignificant with other variables in the model (that is, if the partial F statistic becomes nonsignificant).

Two data sets were developed:

(1) Four seasons, altitude^k ($k = 1, 2, 3$), latitude^j ($j = 1, 2$), longitude^j ($j = 1, 2$), years (1971-1975),

(2) Set (1) without years.

Since data is collected over a five-year span, set (2) was developed to examine the significance of years in a regression model. If the R^2 (percent variation in distribution of percent occurrences of Ri $\leq$ 1 explained by independent variables) using set (1) is not significantly larger than the corresponding values for set (2), we can ignore the yearly variation.

The amounts of variation in turbulence are increased by including cross-product terms of altitude, latitude and longitude. The increase in percent variation accounted for ranges from 3-14%. Examination of residuals (actual percent occurrences of Richardson number $\leq$ 1 minus predicted percent from the regression including cross product) indicated that improvements in the model could be made

in the 8-13 km range. This was accomplished by the inclusion of two new variables quantifying season and year. The percent variation explained by the independent variables increased from 53% to 60% simply by adding the two terms.

5. CHARACTERISTICS OF THE VERTICAL PROFILES OF OCCURRENCES OF Ri ≤ 1

Pronounced seasonal and latitudinal variations are evident in the height profiles of percent occurrences of Ri ≤ 1 shown in Figures 7 through 27. The general features in the height profiles include a sharp cutoff in the occurrences of turbulence in the proximity of the tropopause level and the varying degree of turbulence near or just above the planetary boundary layer (above 2 km). The latter feature depends, of course, on the roughness of terrain at the earth's surface.

At latitudes below approximately 40° N latitude, the contrast between summer and winter occurrences is evident. At the lowest latitudes, near 25° N, the summer occurrences are as low as 5% while winter approaches 40%. The spring and fall values are generally about equal to one another and lie between those of winter and summer. As latitude increases northward, the number of occurrences for all seasons converges to approximately the same values. Another main characteristic in the height profiles of occurrences in turbulence is a major peak in occurrence near the 10 km altitude level. This peak occurs in all seasons but is much more pronounced in the winter occurrences at low latitudes (40%) and is lowest in summer at low latitudes (10%).

For stations near mountain ranges, the percent occurrences increase above the boundary layer. The degree of turbulence above the boundary layer appears to be related to the height of the mountain ranges in the proximity of the data recording stations. Height profiles obtained from data at these stations are characteristically quite different from those obtained from data at stations located in relatively flat regions. Effects from the mountains appear to dominate the height profiles of occurrence of turbulence to the degree that seasonal features as well as the 10 km peak are no longer evident. This effect is apparent in the profiles of Glasgow, Montana (Figure 12), Great Falls, Montana (Figure 13) and Denver, Colorado (Figure 10) where the highest mountains within several tens of miles of the station location are, respectively, 3 K ft, 7 K ft and 9 K ft. The altitude of turbulence occurrence increases with height of surrounding mountains.

6. CONCLUSIONS

Rawinsonde wind and temperature data have been used to determine Richardson's numbers for 1 km levels between 2 km and 25 km. A critical value of the gradient Richardson number of one has been used as an indicator of the likelihood of occurrence of turbulence. The patterns in the occurrence of turbulence have been investigated for the seasons and years from 1971-1975. A regression analysis performed on data from 21 rawinsonde stations located within the continental United States, indicated that:

(1) Yearly variations in the occurrences of $Ri \leq 1$ were found to be small; these contributed, at most, less than 1% of the total variation.

(2) The amounts of variation in turbulence explained by knowing latitude, longitude, altitude and season included 38% in the 2-7 km bin, 60% in the 8-13 bin, 42% in the 14-19 km bin and 11% in the 20-25 km bin.

(3) The patterns of occurrence of turbulence based on a critical Richardson number are stable and reproducible from year to year.

(4) Examination of the vertical profiles of occurrences of $Ri \leq 1$ reveal that there are pronounced latitudinal and seasonal variations. For the lower latitude data (~ 25° N), the summer occurrences are as low as 5% while winter occurrences approa 40%. Spring and fall occurrences are about equal and lie between winter and summer extremes. As latitude increases northward, the number of occurrences for all seasons converges to approximately the same values.

(5) A peak in occurrence near the 10 km altitude level is found in the height profiles from low to mid-latitudes. The height profiles of occurrences of $Ri \leq 1$ are markedly different for stations in the vicinity of mountain ranges.

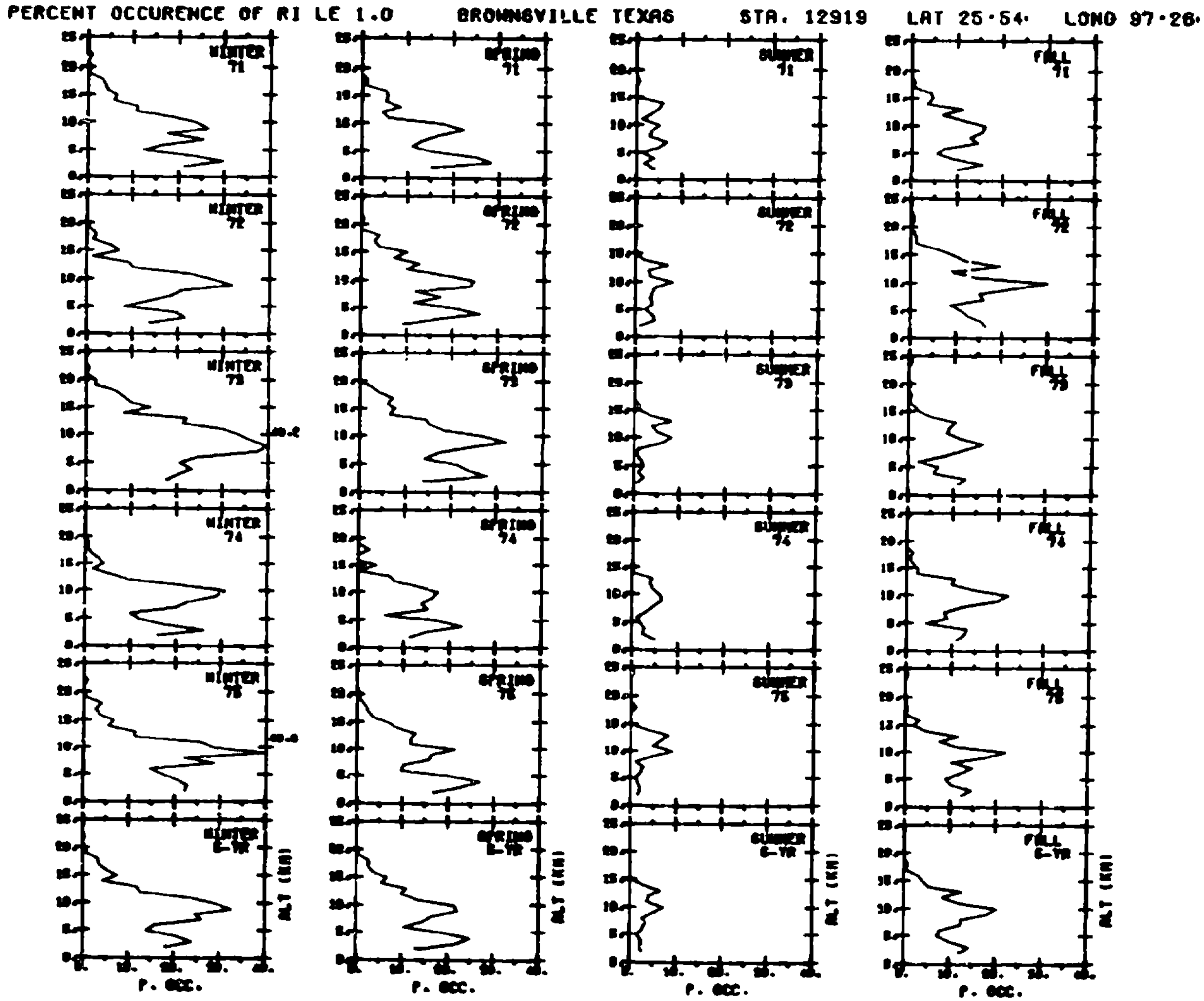

Figure 7. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Brownsville, Texas Station Used in the Regression Analysis

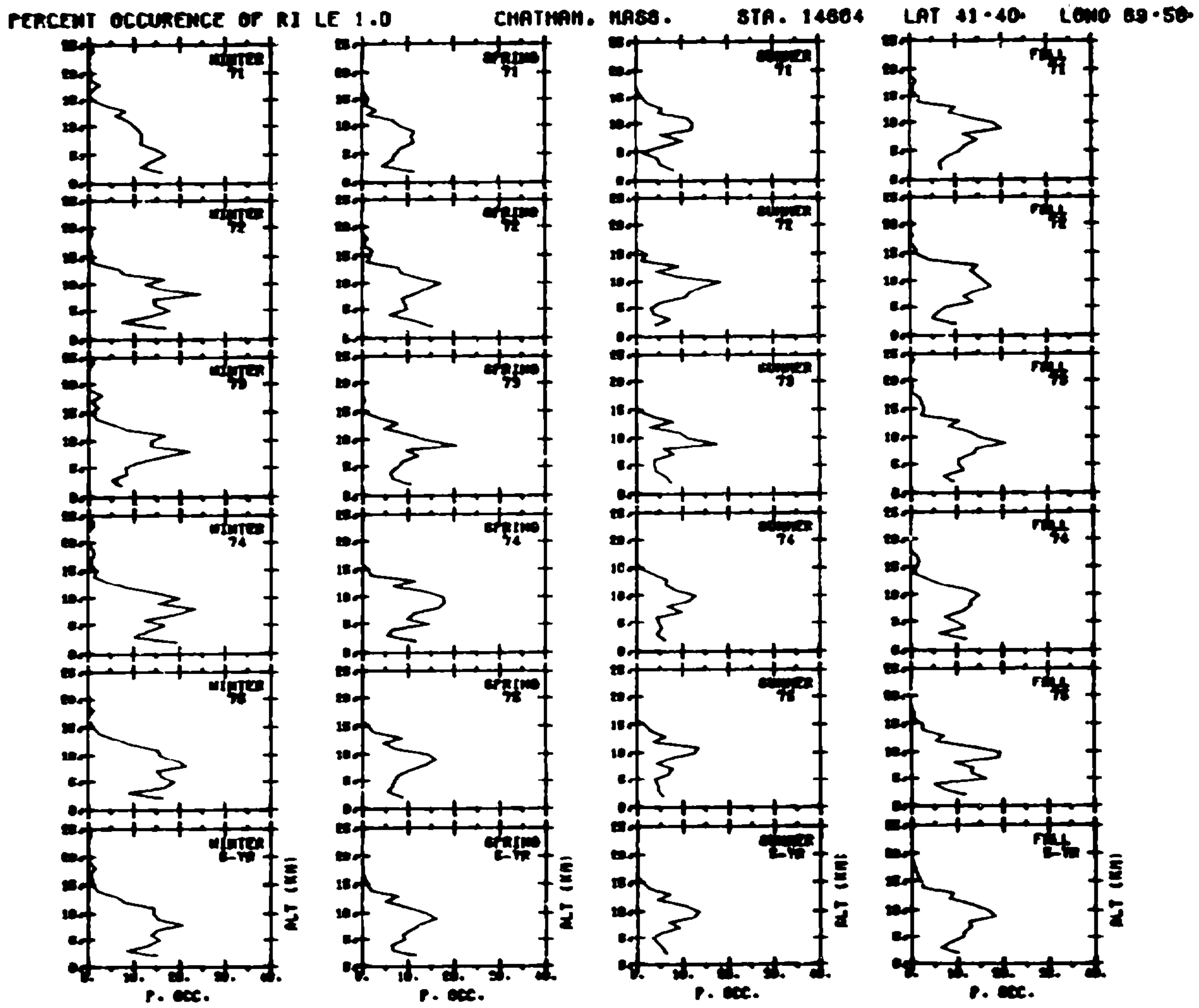

Figure 8. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Chatham, Mass. Station Used in the Regression Analysis

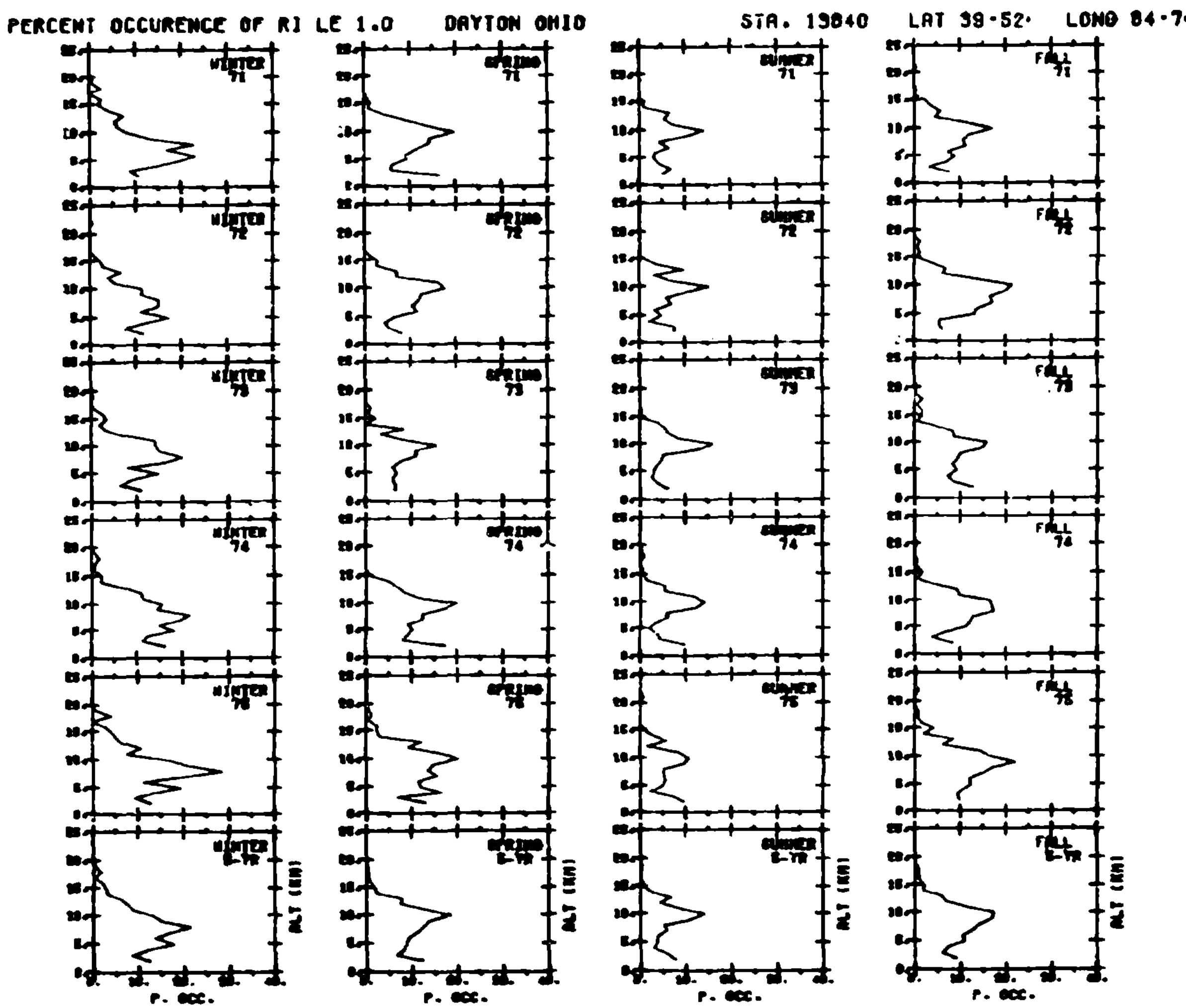

Figure 9. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Dayton, Ohio Station Used in the Regression Analysis

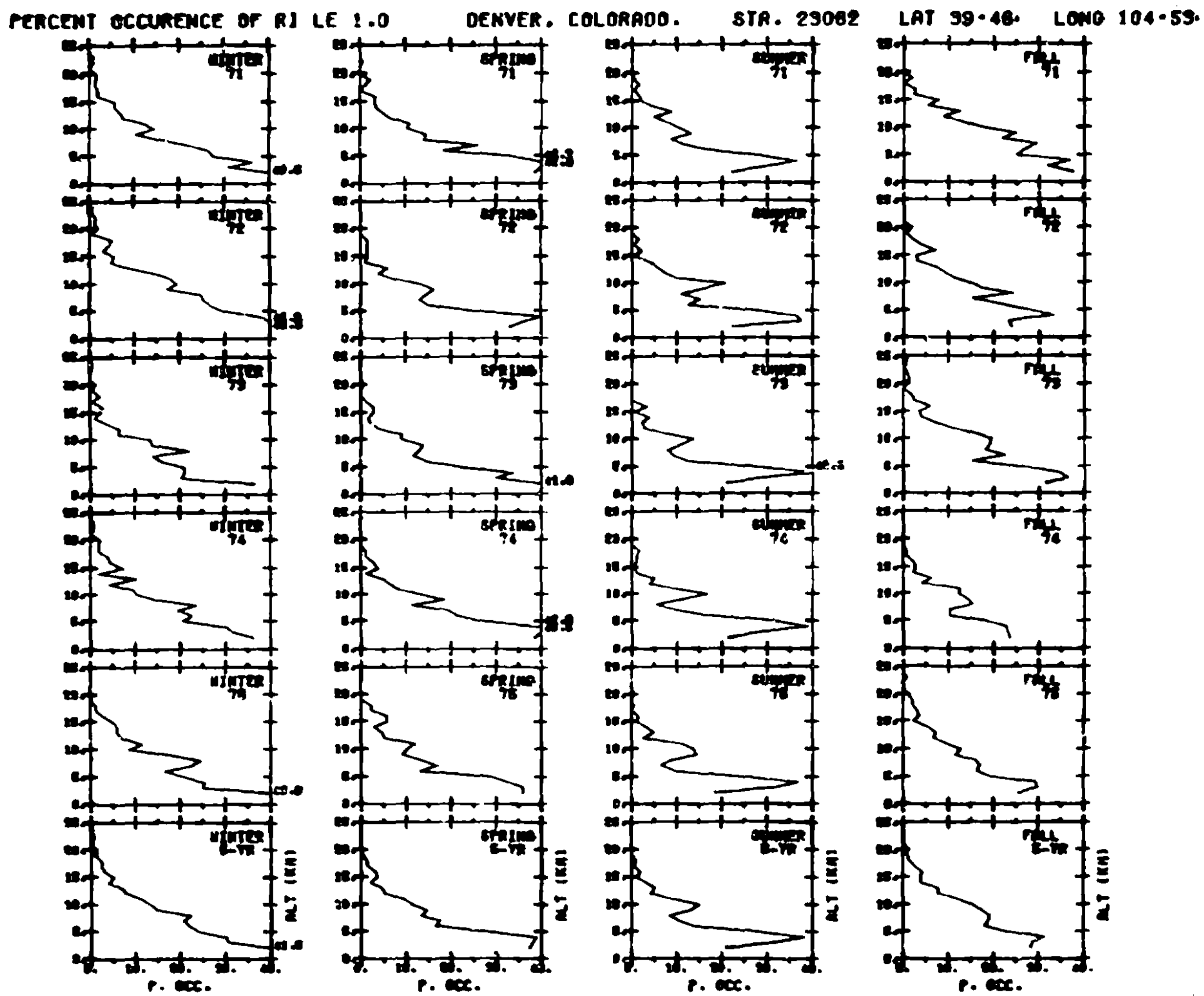

Figure 10. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Denver, Colo. Station Used in the Regression Analysis

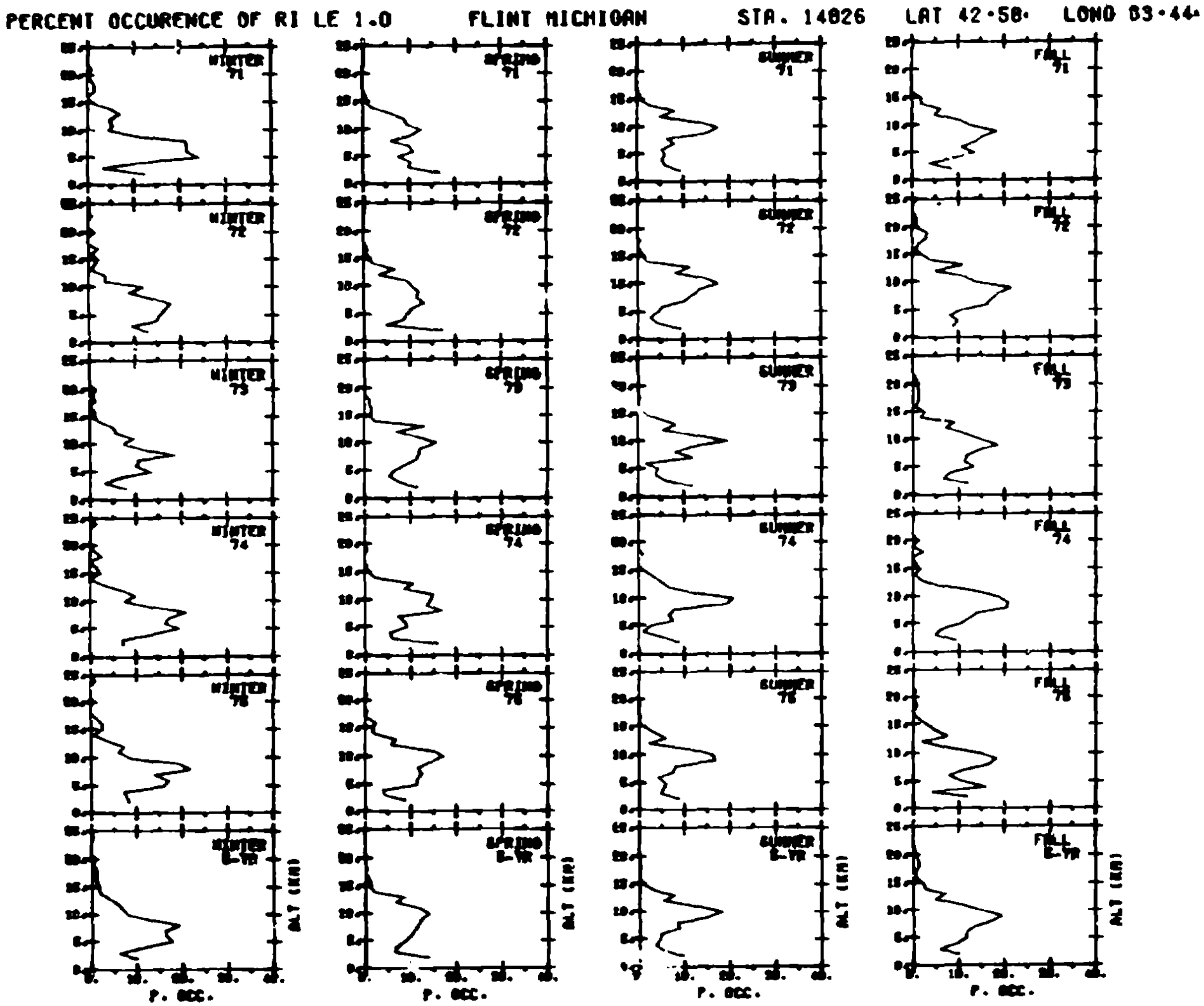

Figure 11. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Flint, Mich. Station Used in the Regression Analysis

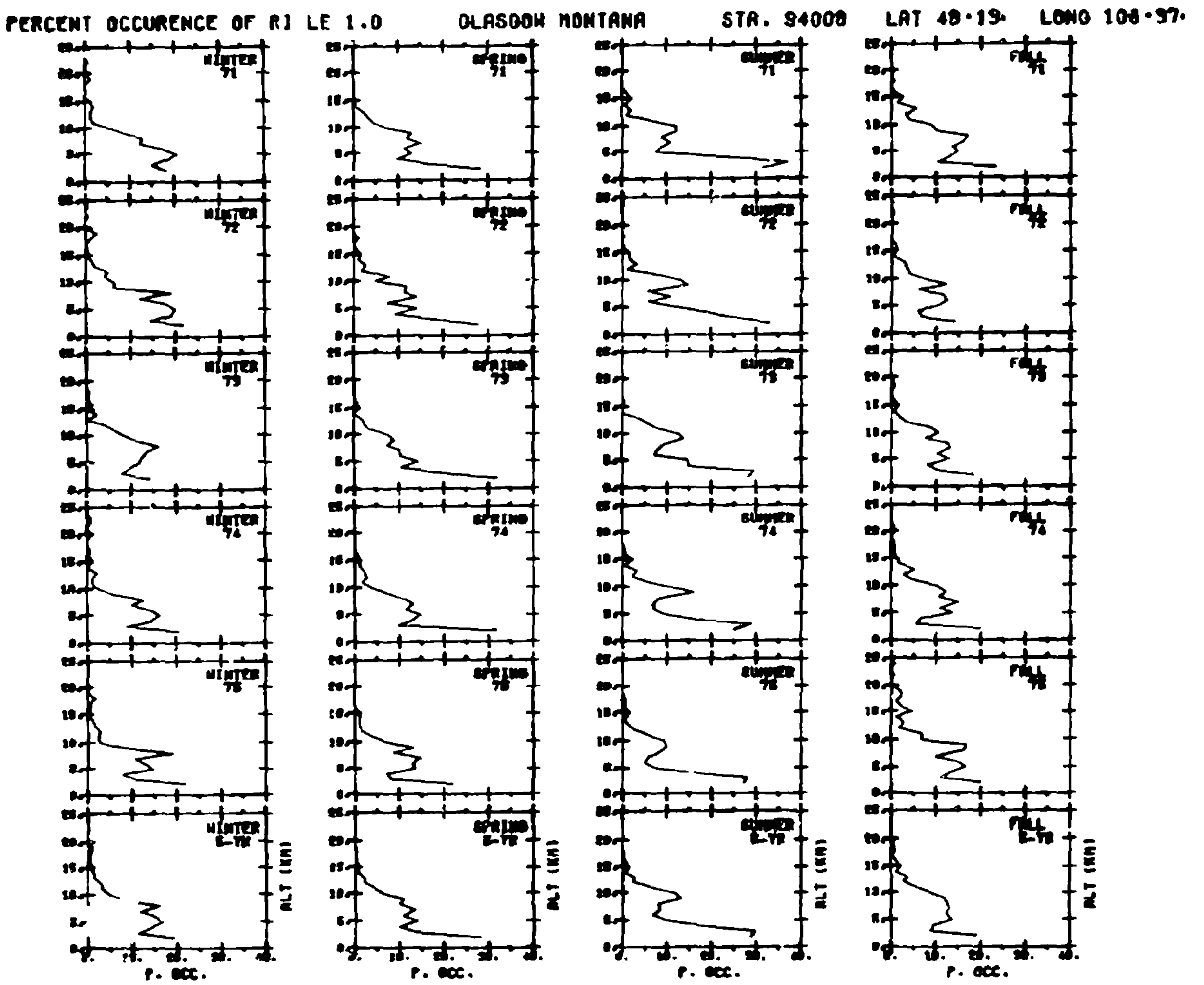

Figure 12. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Glasgow, Mont. Station Used in the Regression Analysis

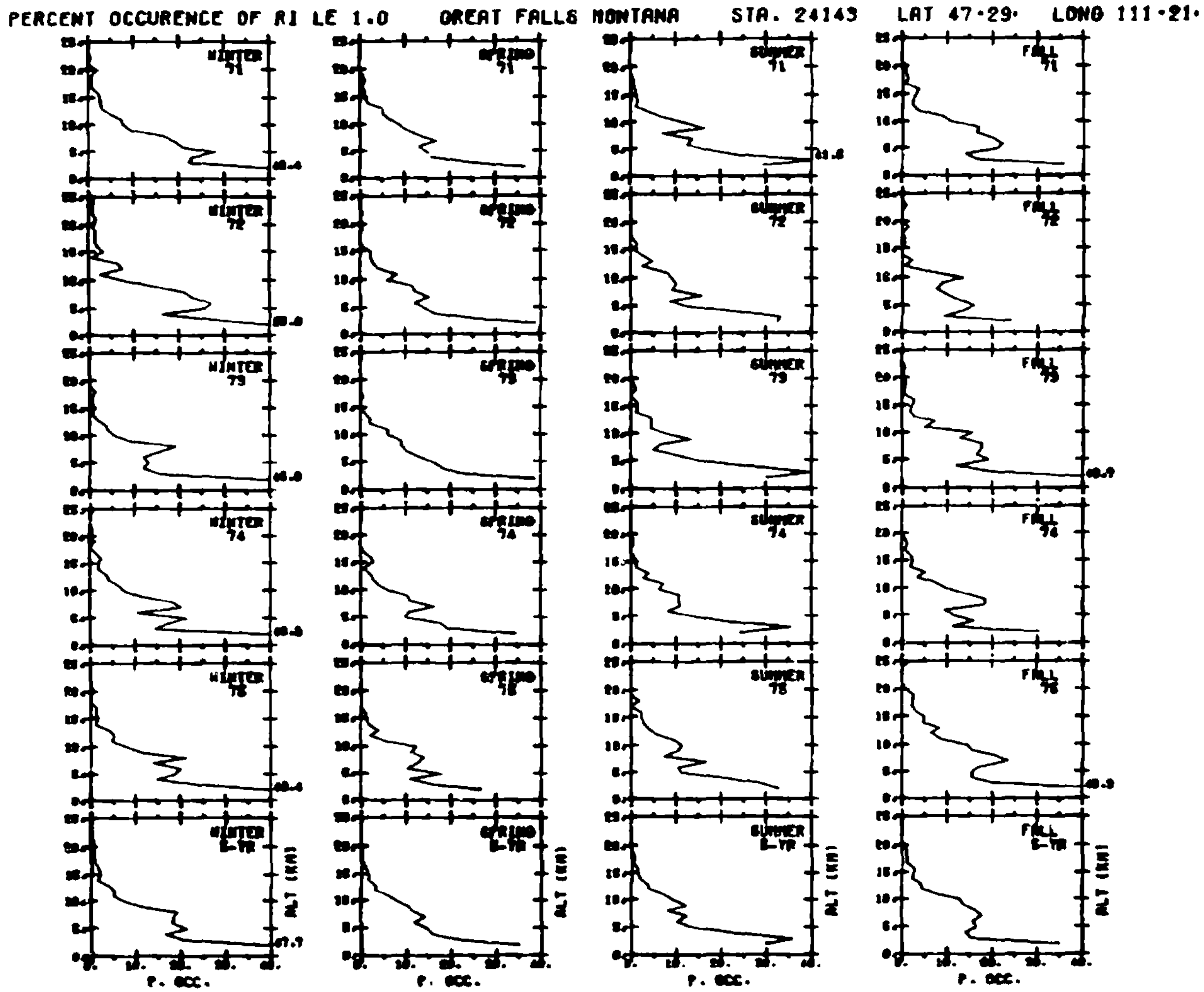

Figure 13. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Great Falls, Mont. Station Used in the Regression Analysis

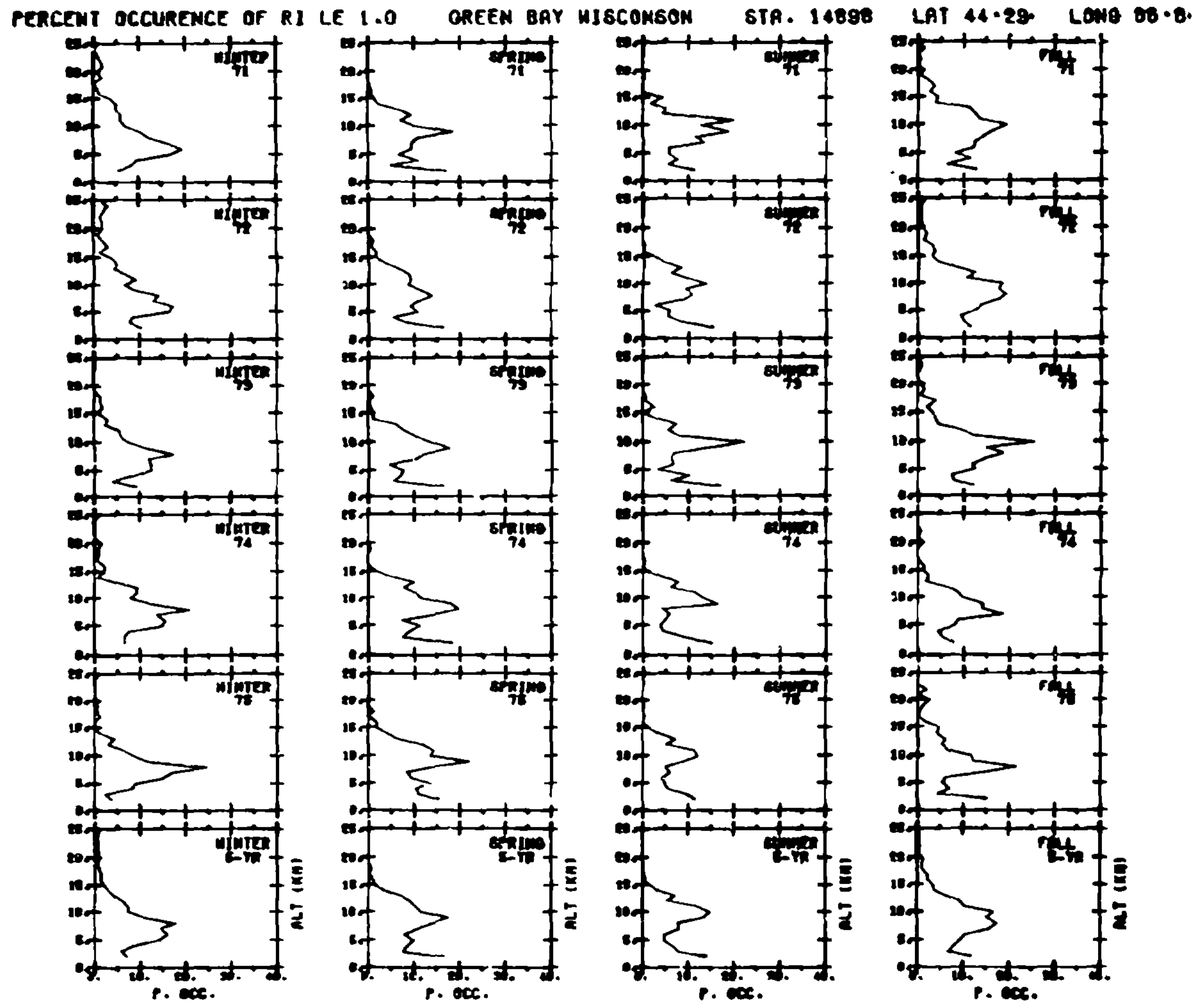

Figure 14. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Green Bay, Wisc. Station Used in the Regression Analysis

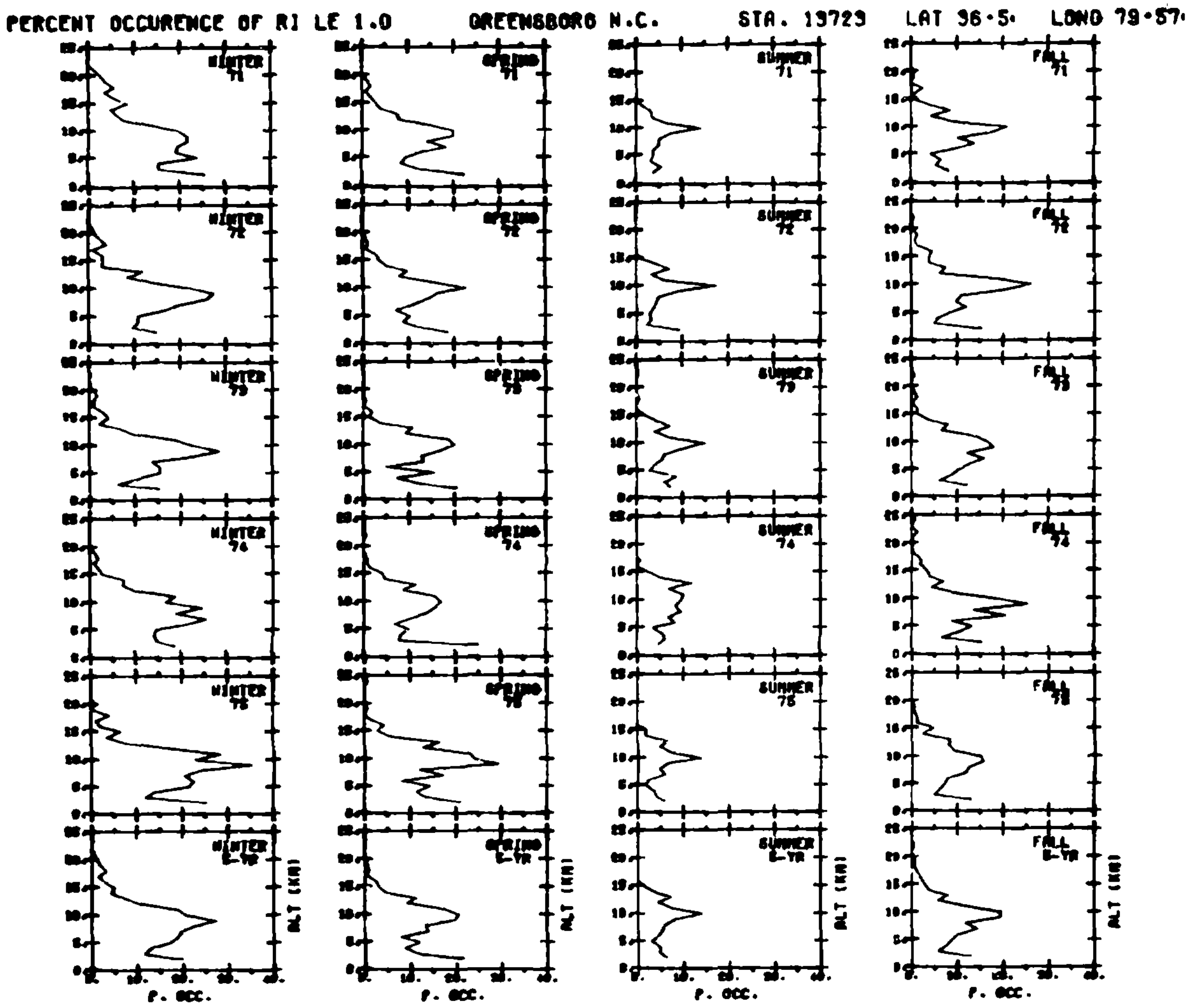

Figure 15. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Greensboro, N.C. Station Used in the Regression Analysis

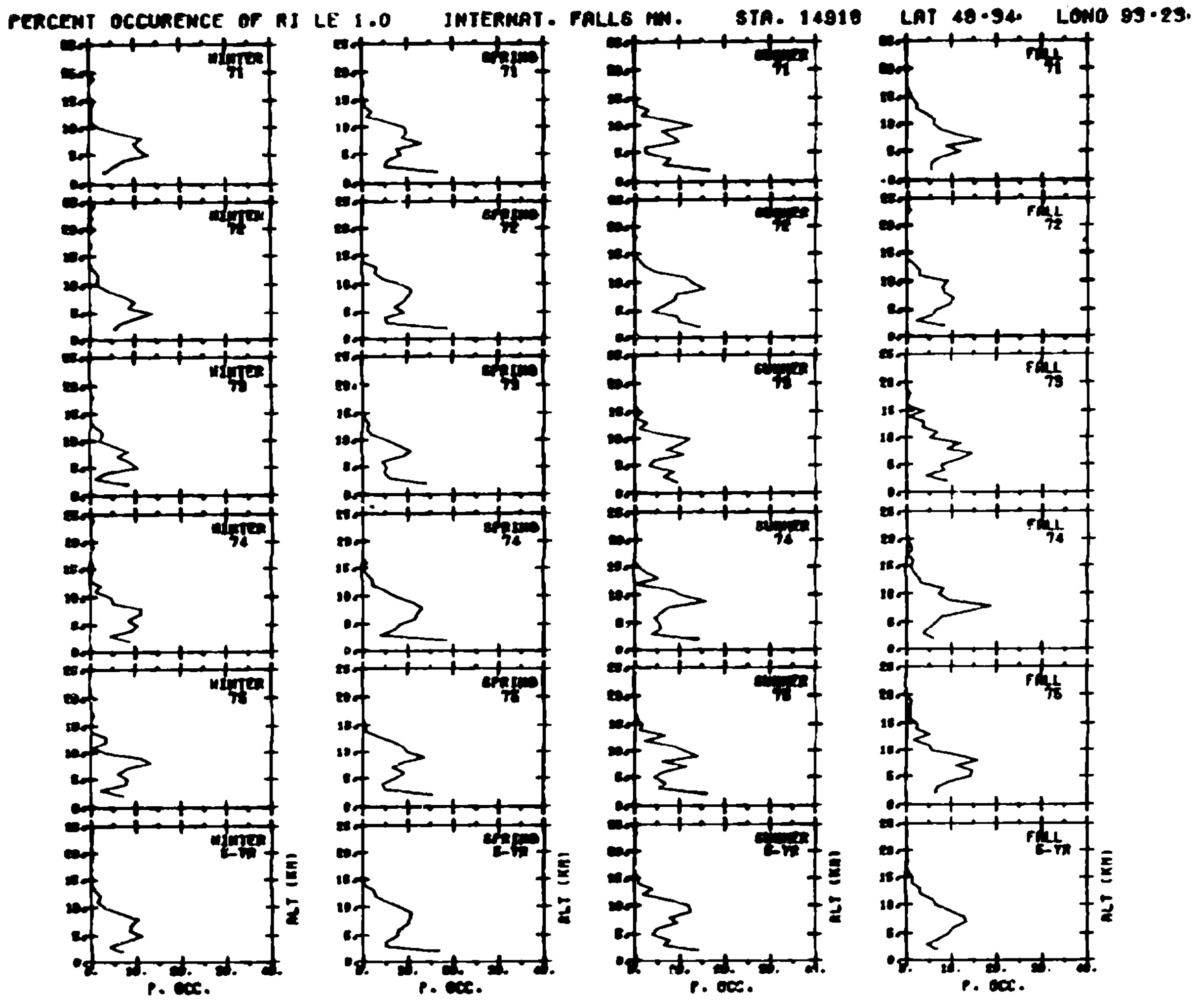

Figure 16. Seasonal Height *Profiles of* Occurrences at *Ri* ≤ *1 for* International Falls, MN Station Used in the Regression Analysis

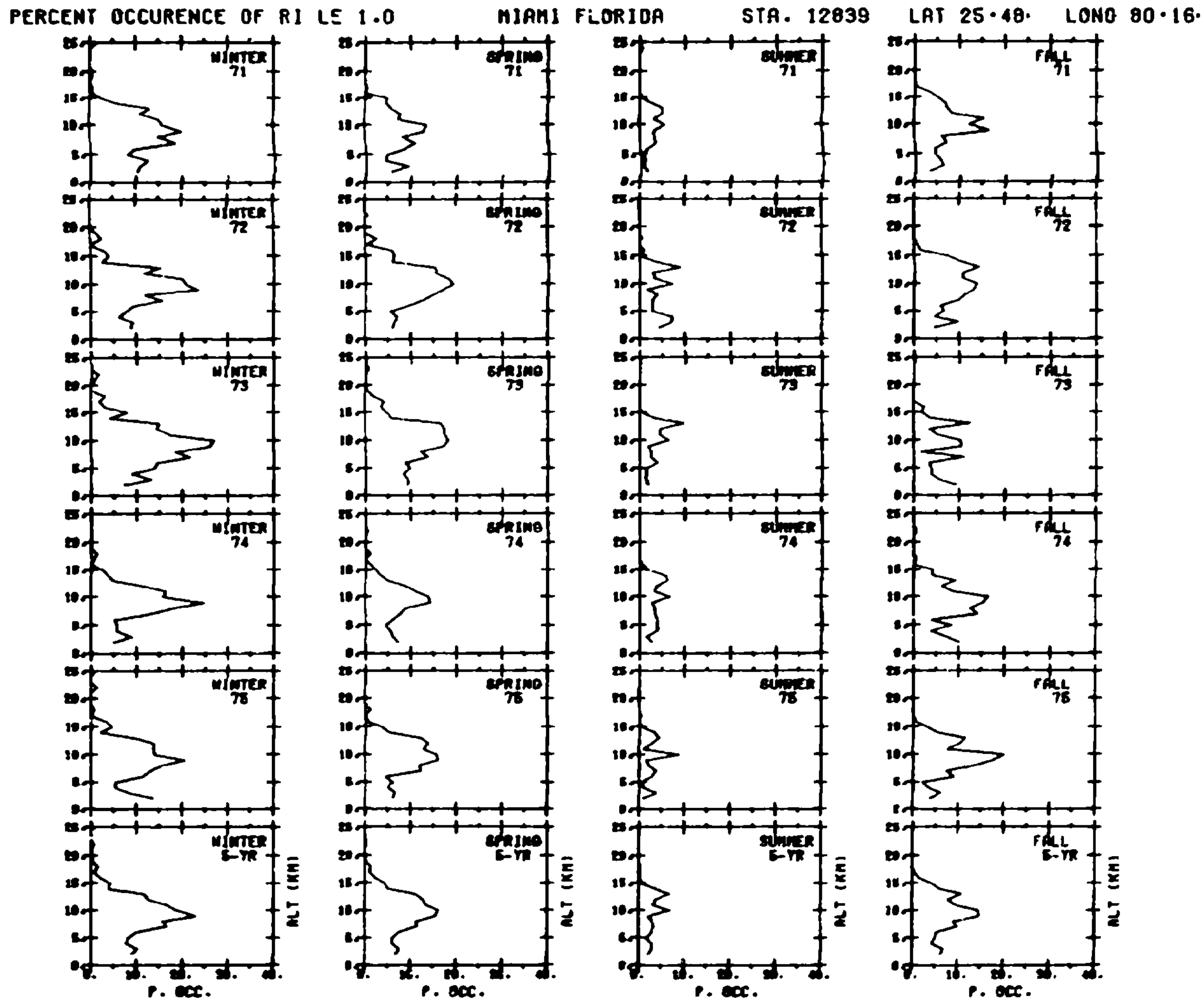

Figure 17. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Miami, Fla. Station Used in the Regression Analysis

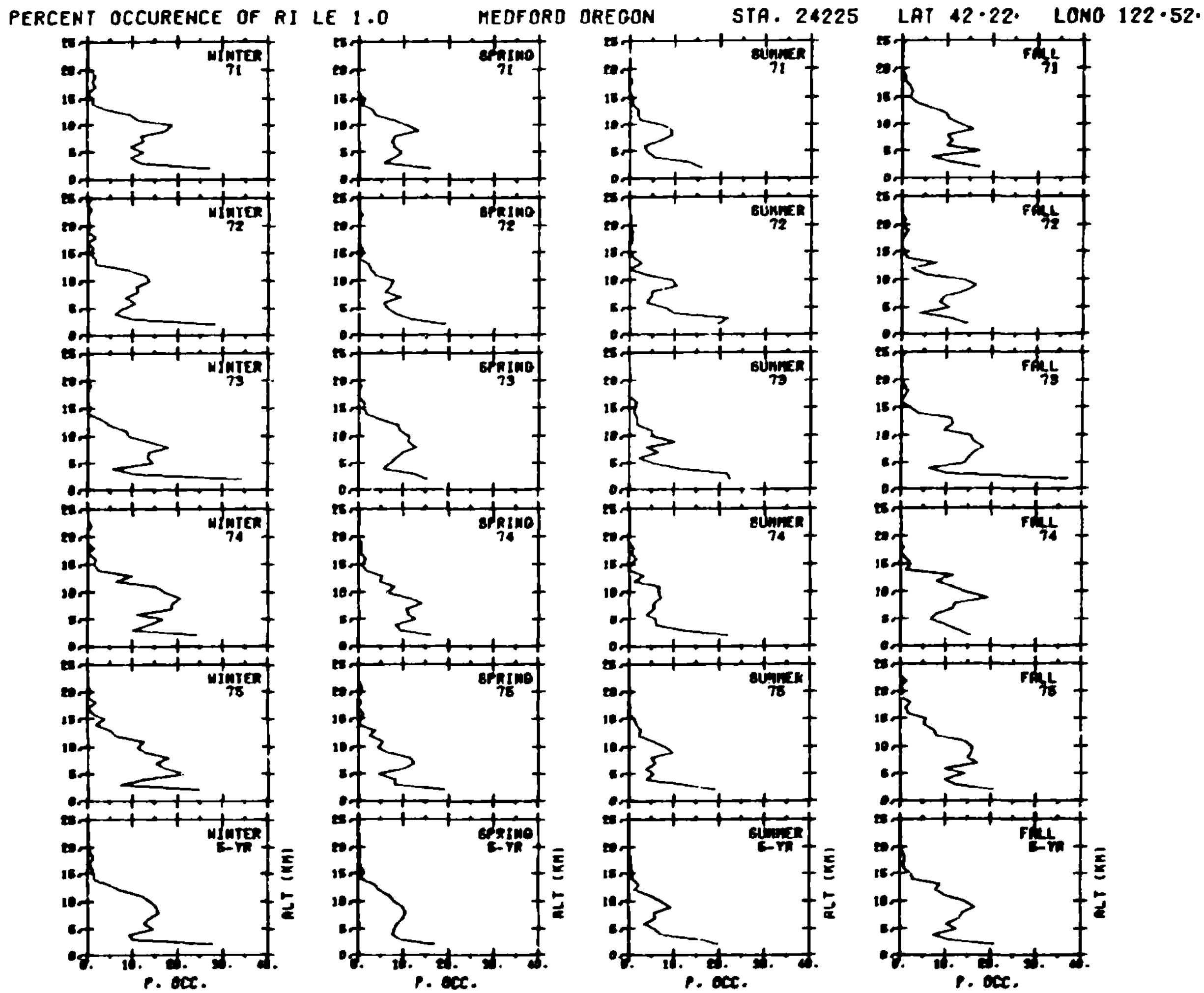

Figure 18. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Medford, Oregon Station Used in the Regression Analysis

PERCENT OCCURENCE OF RI LE 1.0 NORTH PLATTE NEB. STA. 24023 LAT 41·8· LONG 100·41·

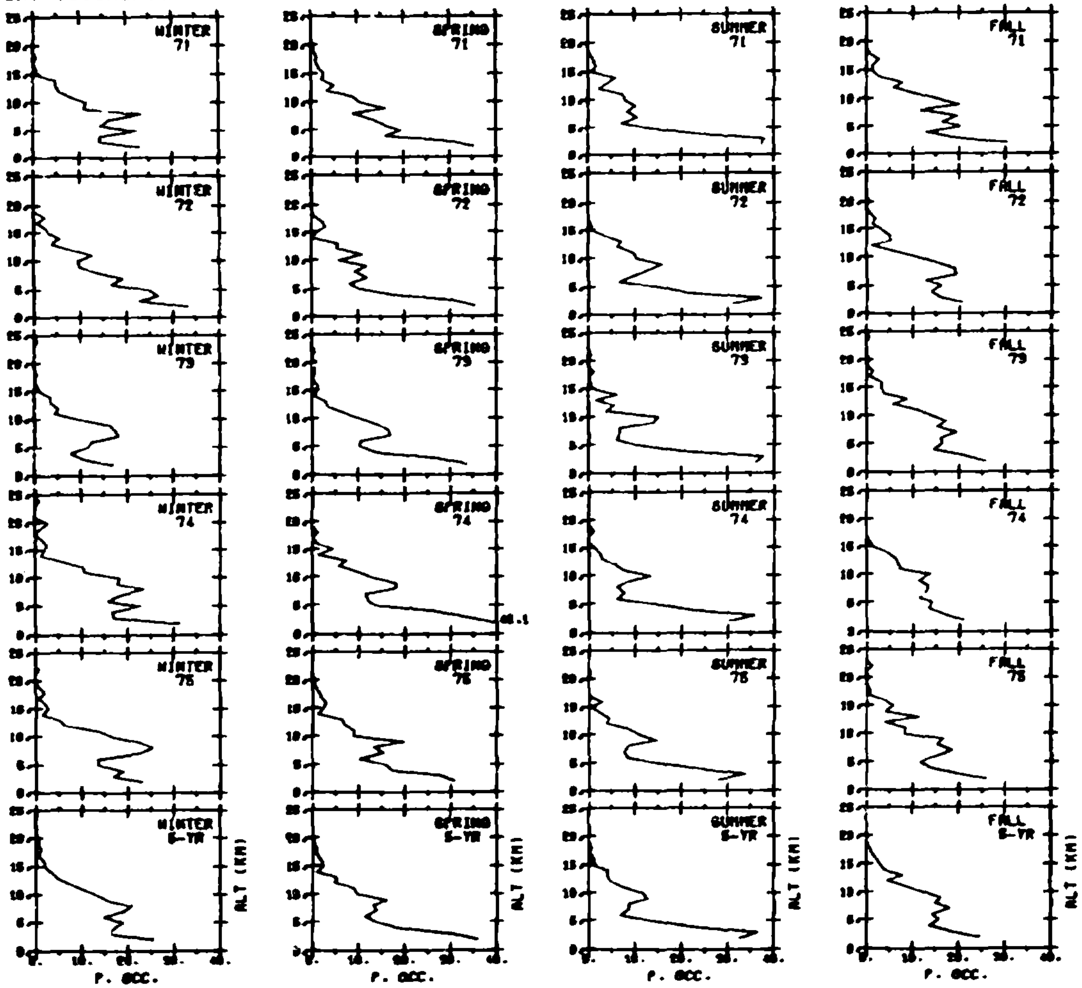

Figure 19. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for North Platte, Neb. Station Used in the Regression Analysis

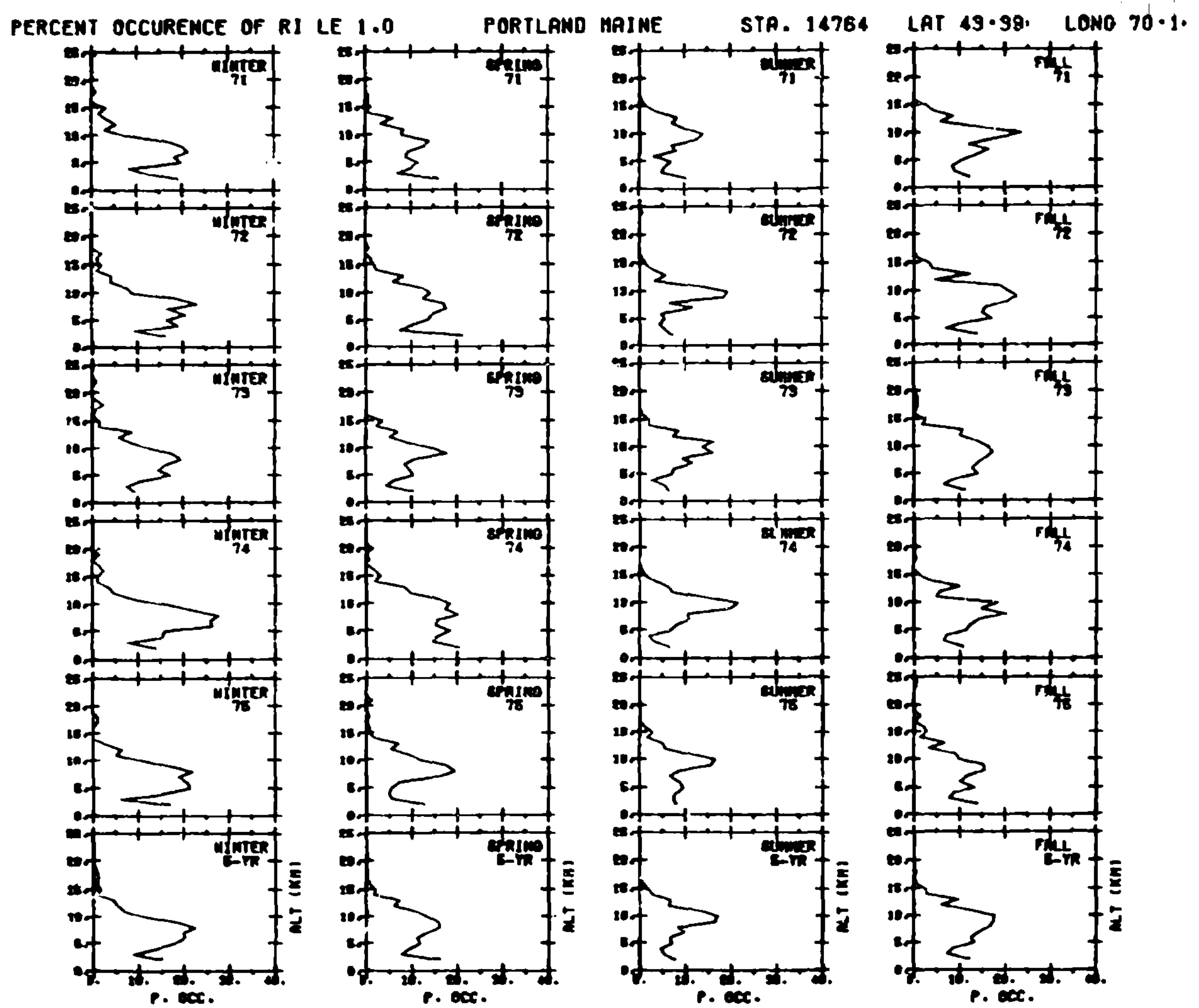

Figure 20. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Portland, Me. Station Used in the Regression Analysis

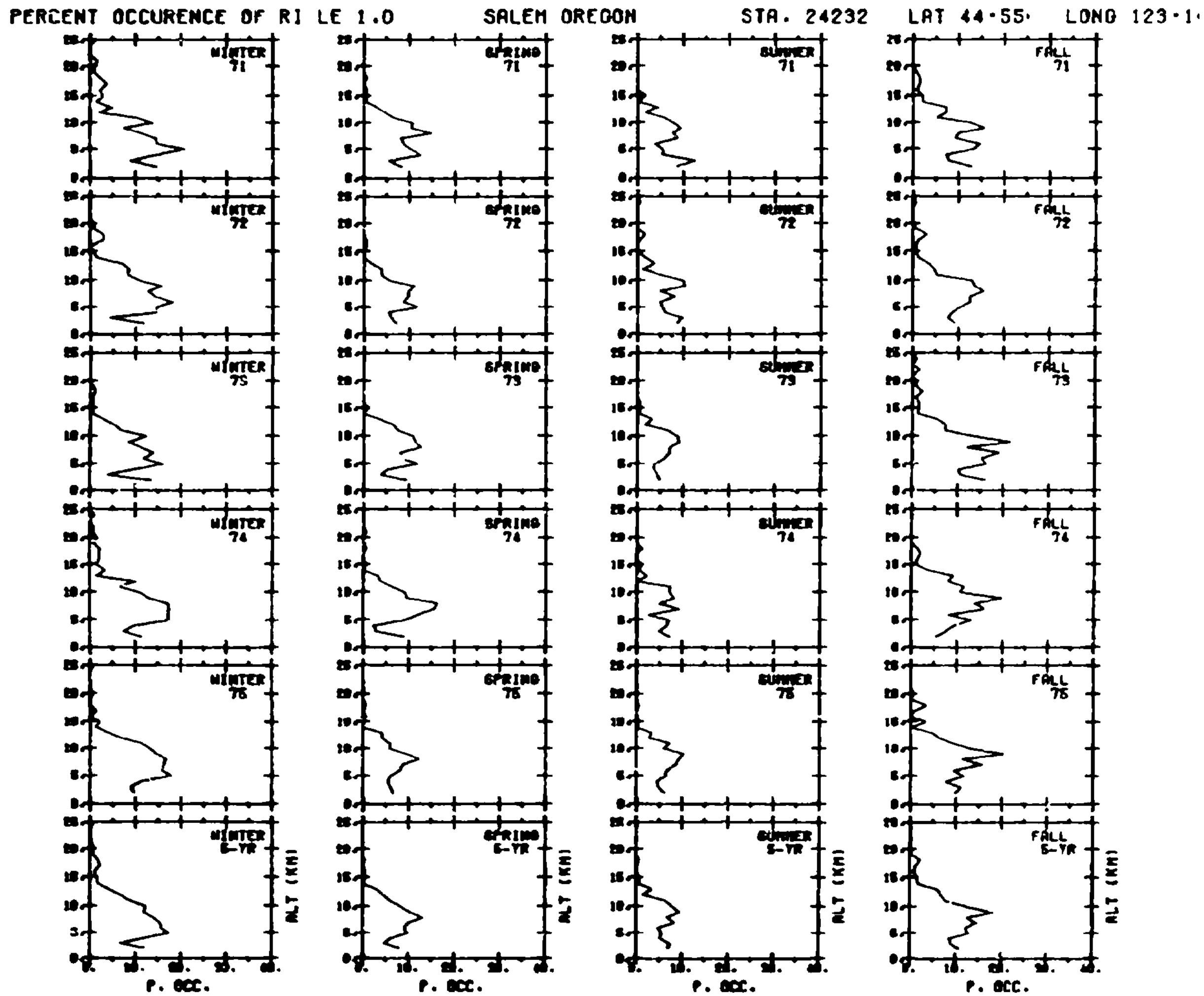

Figure 21. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Salem, Oregon Station Used in the Regression Analysis

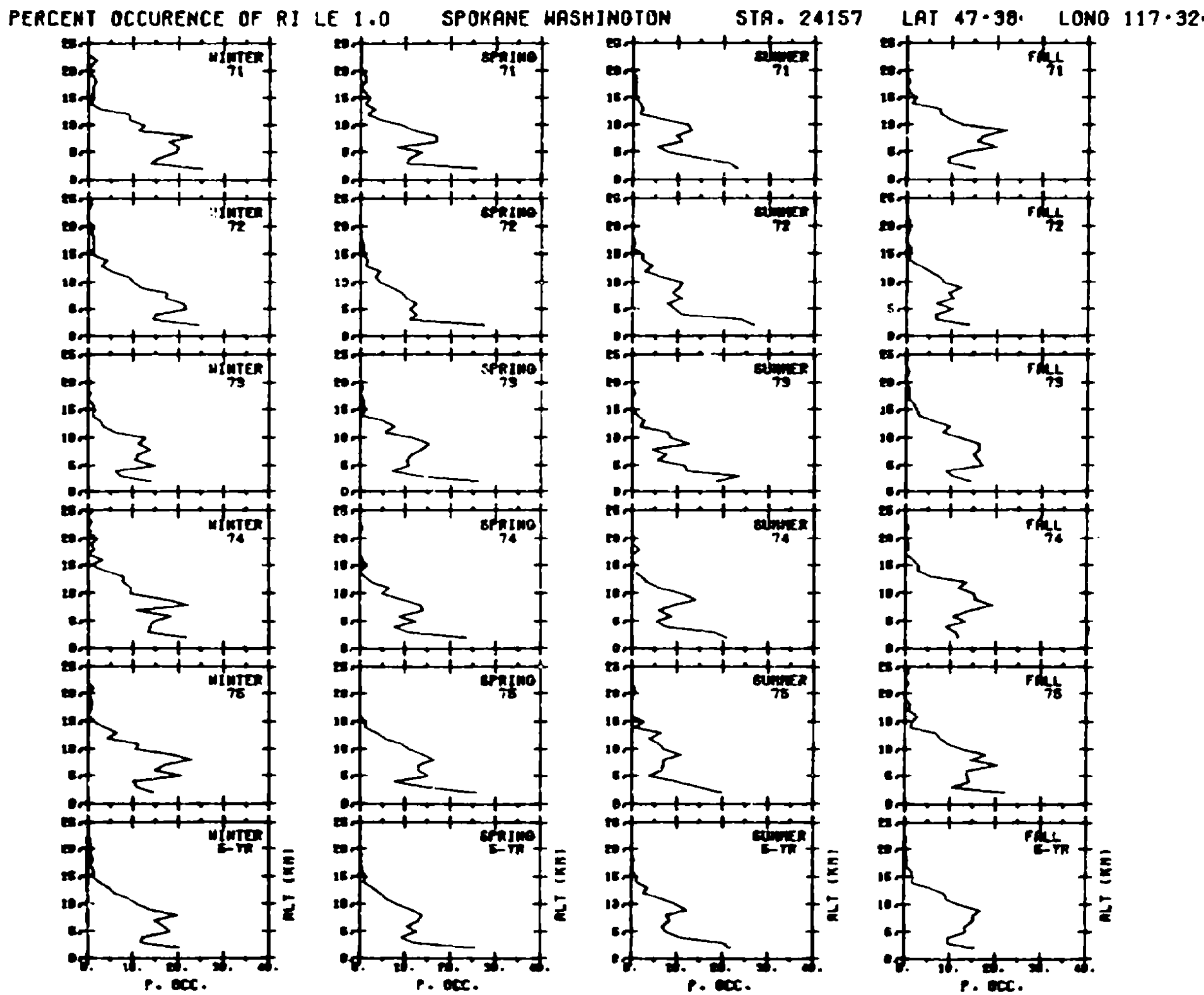

Figure 22. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Spokane, Wash. Station Used in the Regression Analysis

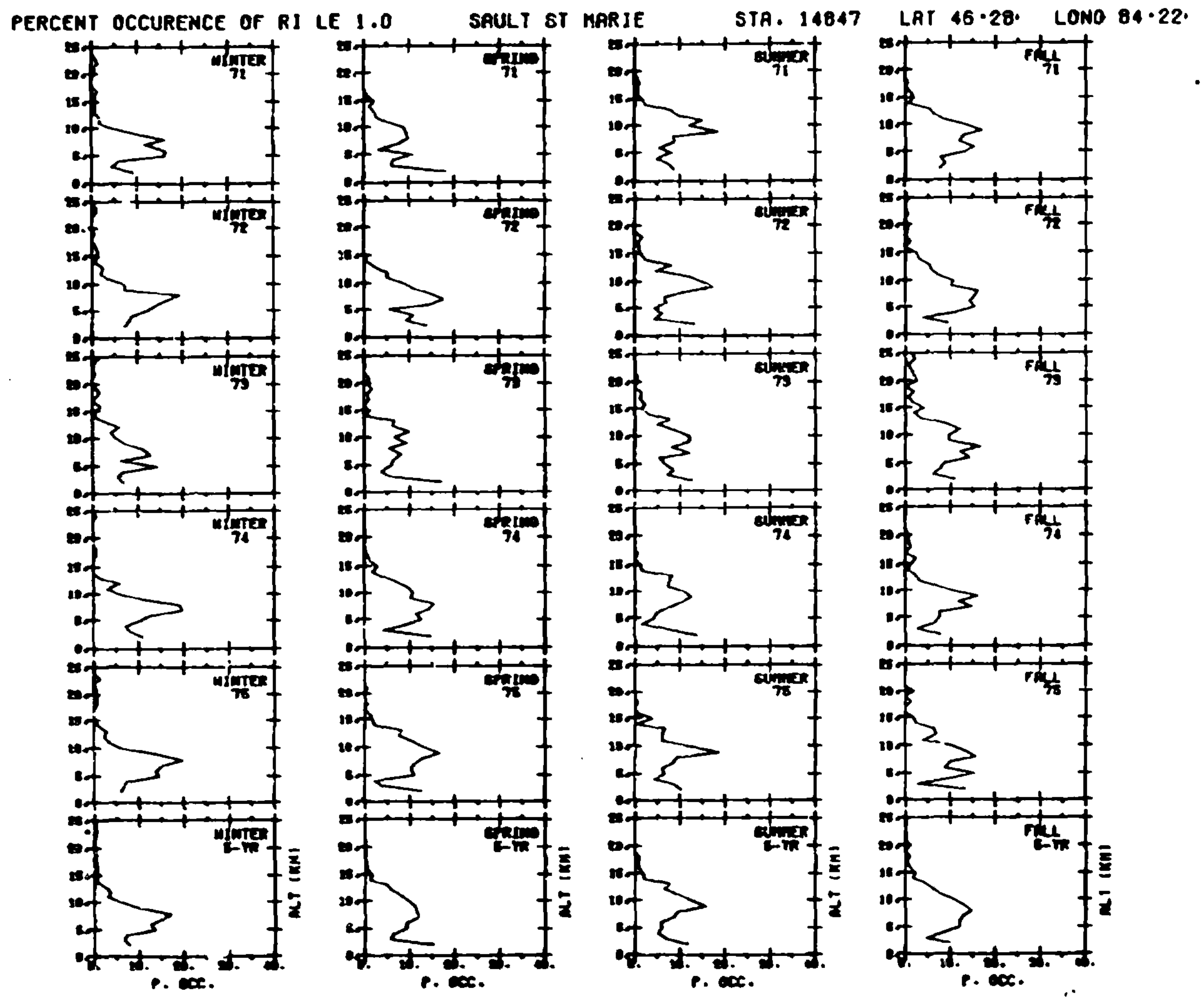

Figure 23. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Sault Ste. Marie, Mich. Station Used in the Regression Analysis

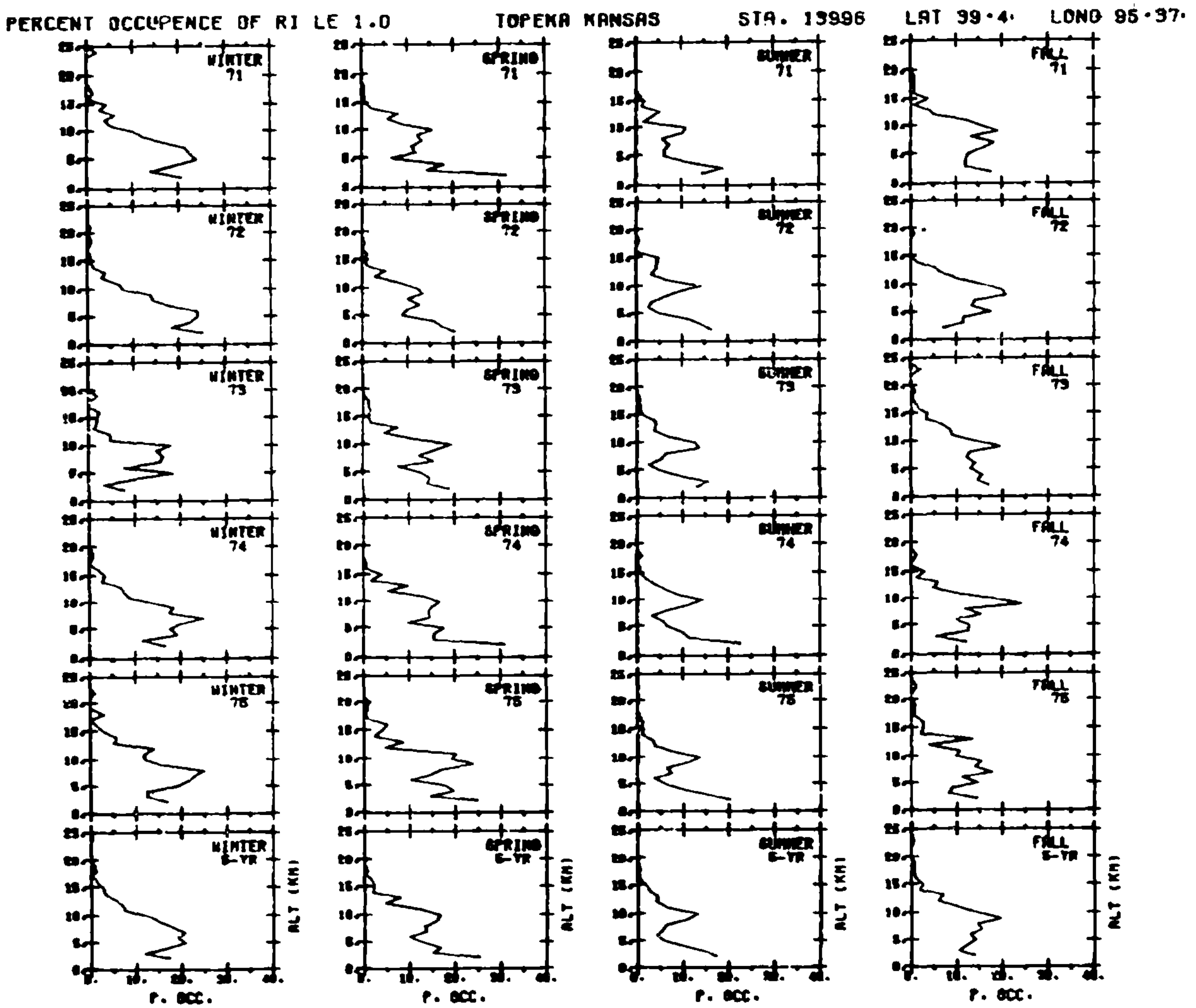

Figure 24. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Topeka, Kansas Station Used in the Regression Analysis

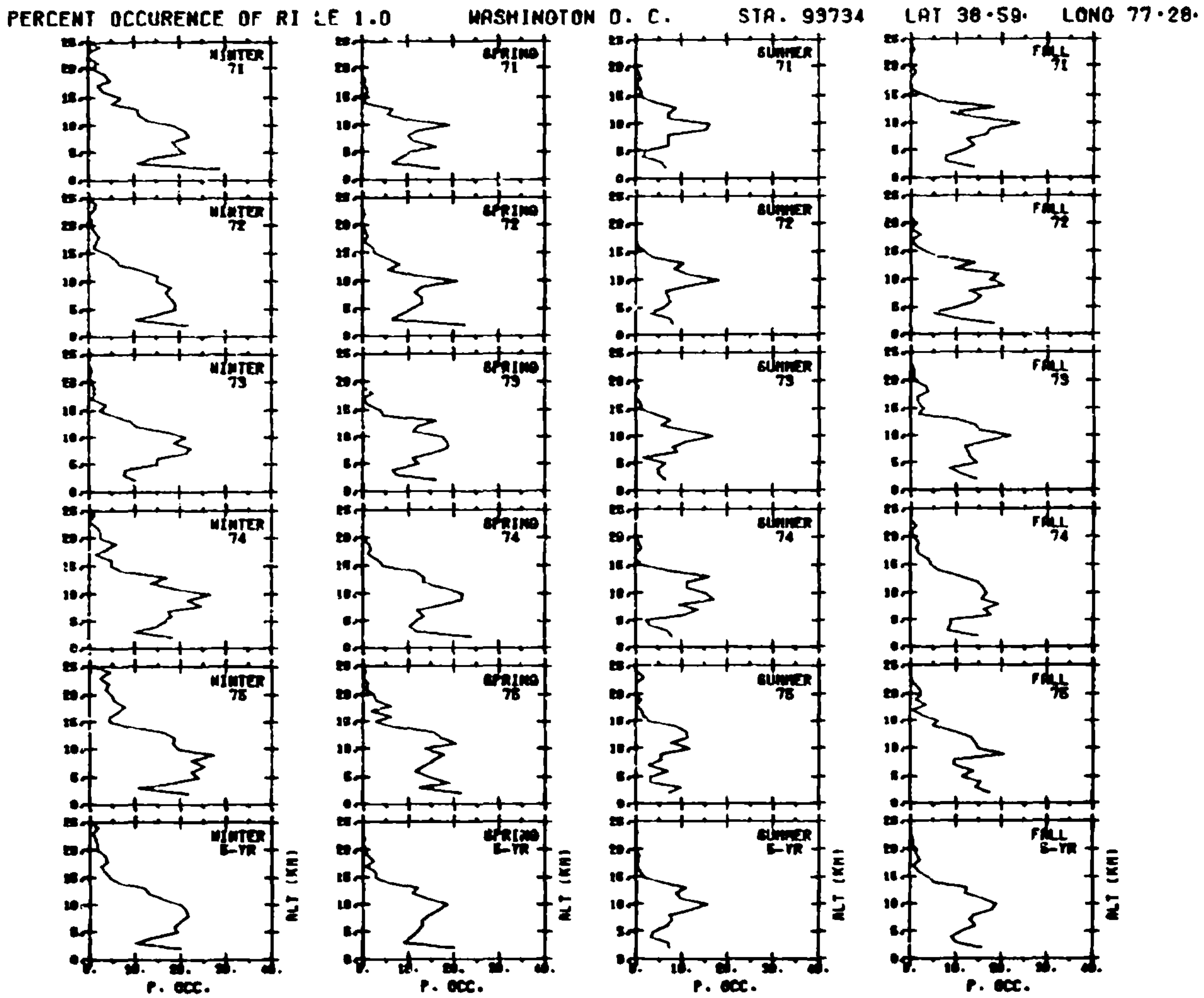

Figure 25. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Washington, D.C. Station Used in the Regression Analysis

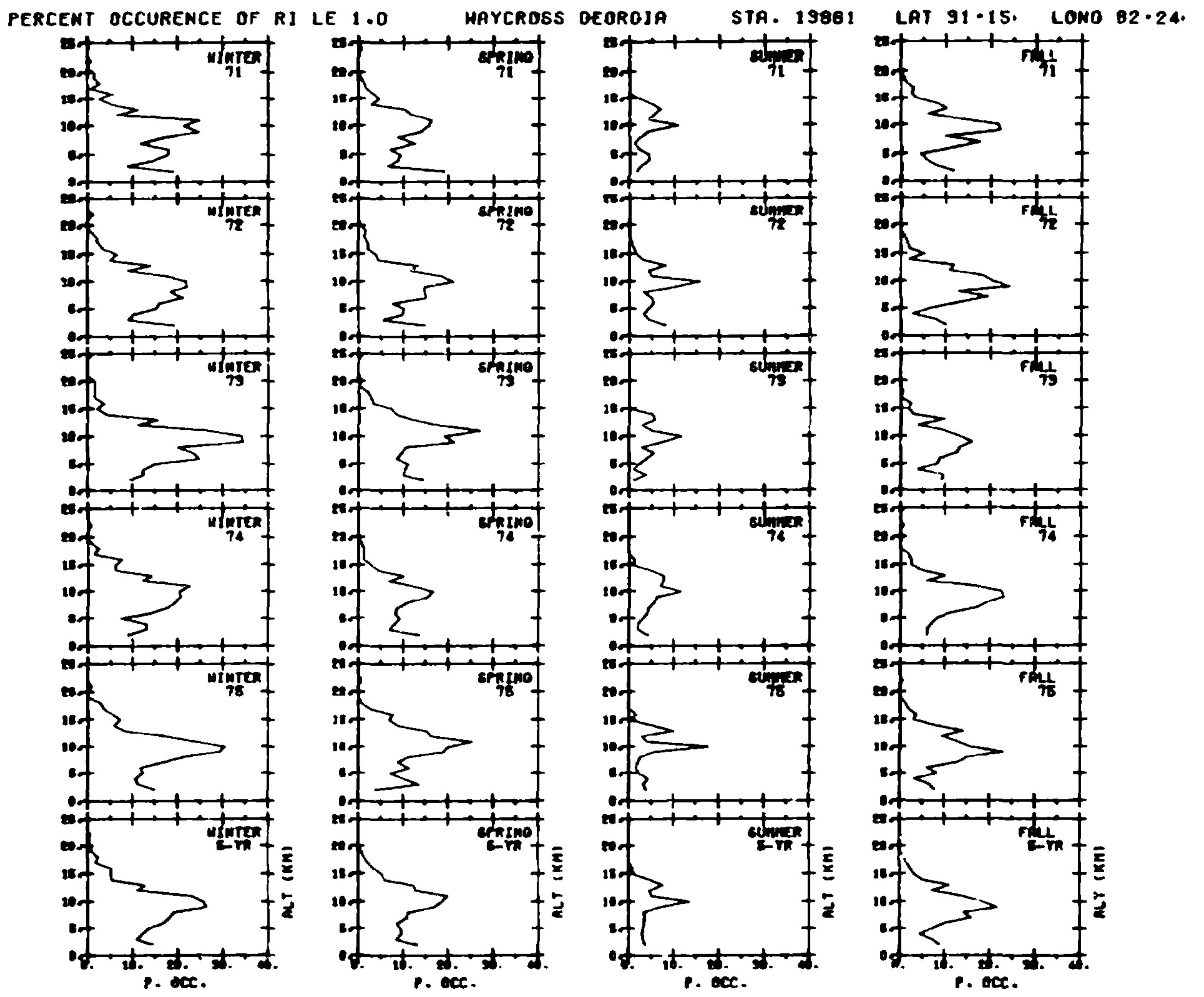

Figure 26. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Waycross, Ga. Station Used in the Regression Analysis

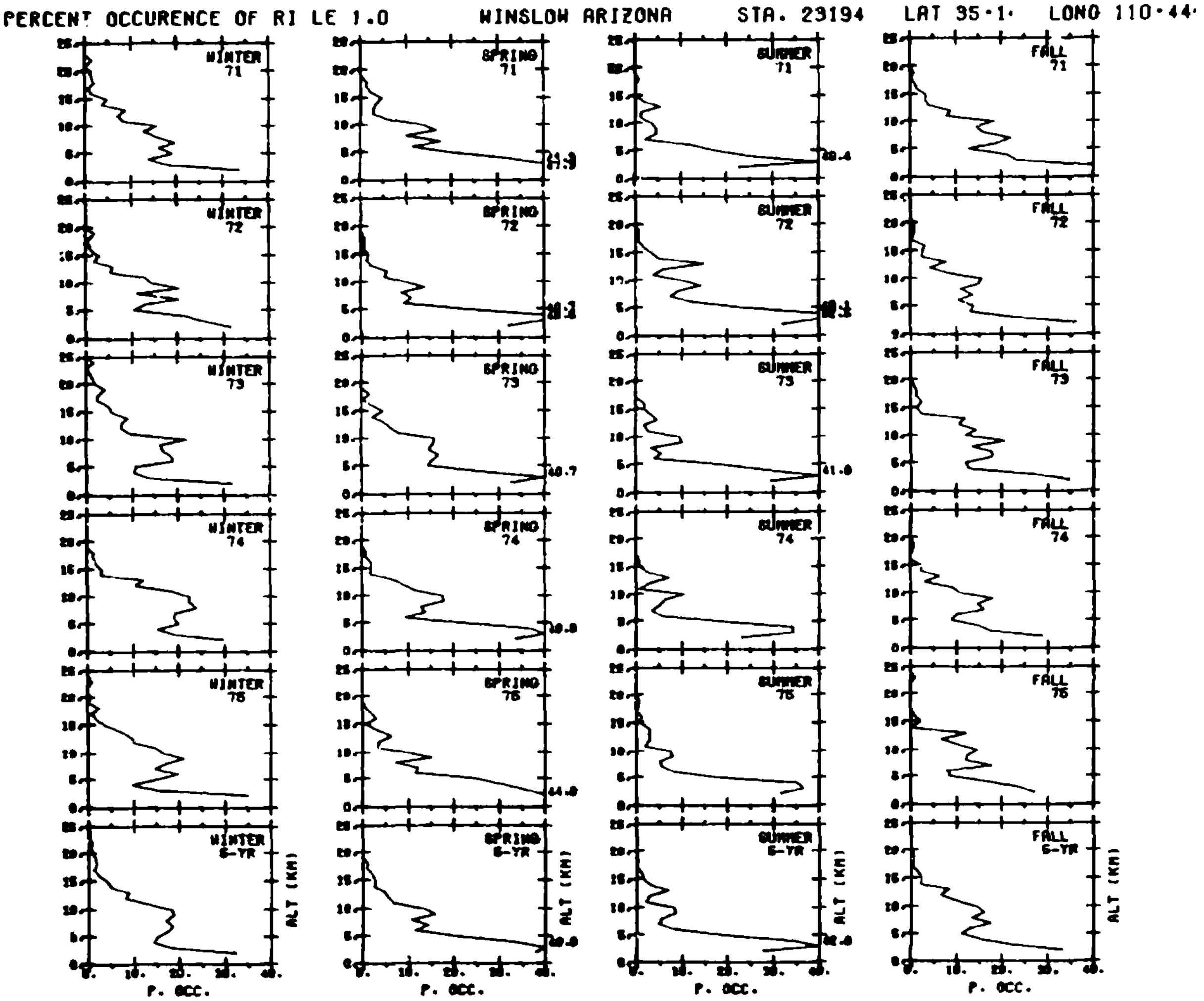

Figure 27. Seasonal Height Profiles of Occurrences at Ri ≤ 1 for Winslow, Ariz. Station Used in the Regression Analysis

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